OXFORD IB DIPLOMA PROGRAMME

WORKED SOLUTIONS

MATHEMATICAL STUDIES STANDARD LEVEL

COURSE COMPANION

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1 Number and algebra 1

Answers

Skills check

1 **a** $y = 3x^2(x-1)$

$y = 3(-0.1)^2(-0.1-1)$

$y = -0.033$

b $y = \frac{(x-1)^2}{x}$

$y = \frac{(-0.1-1)^2}{-0.1}$

$y = -12.1$

c $y = (1-x)(2x+1)$

$y = (1-(-0.1))(2\times -0.1+1)$

$y = 0.88$

2 **a** $3x - 7 = 14$

$3x = 14 + 7$

$x = \frac{21}{3}$

$x = 7$

b $2(x-6) = 4$

$x - 6 = \frac{4}{2}$

$x = 2 + 6$

$x = 8$

c $\frac{1}{2}(1-x) = 0$

$1 - x = 0$

$x = 1$

d $x \cdot x = 16$

$x = 4$ or $x = -4$

3 **a** 8% of 1200 $= \frac{8}{100} \times 1200 = 96$

b 0.1% of 234 $= \frac{0.1}{100} \times 234 = 0.234$

4 **a** $10 - x \le 1$

$9 \le x$

b $3x - 6 > 12$

$x > \frac{18}{3}$

$x > 6$

c $2x \le 0$

$x \le 0$

5 remember that the absolute value of a number is always *greater than or equal to zero* but never negative.

a $|-5| = 5$

b $\left|\frac{1}{2}\right| = \frac{1}{2}$

c $|5-7| = |-2| = 2$

d $\left|\frac{12-8}{8}\right| \times 100 = \left|\frac{4}{8}\right| \times 100 = \frac{4}{8} \times 100 = 50$

Exercise 1A

a **i** $2a + b = 2\times 2 + 4 = 8$

ii $2(a+b) = 2(2+4) = 12$

iii $a^2 - b^2 = 2^2 - 4^2 = -12$

iv $(a-b)^2 = (2-4)^2 = (-2)^2 = 4$

b **i** Yes **ii** Yes **iii** No **iv** Yes

Exercise 1B

1 **a** $4x + 2 = 0$

$4x = -2$

$x = \frac{-2}{4}$ (or $x = -0.5$)

b It is not an integer.

2 **a** $x \cdot x = 4$

$x = 2$ or $x = -2$

b Both are integers.

3 **a** **i** $\frac{a-b}{a+b} = \frac{-2-4}{-2+4} = \frac{-6}{2} = -3$

ii $3a^2 - \frac{9}{b} = 3(-2)^2 - \frac{9}{4} = 12 - \frac{9}{4} = \frac{39}{4}$ (or 9.75)

b **i** It is an integer.

ii It is not an integer.

Exercise 1C

1 Look for the decimal expansion of each of the fractions

$\frac{2}{3} = 0.66666\ldots$ Therefore the decimal expansion of this fraction recurs.

$-\frac{5}{4} = -1.25$. Therefore the decimal expansion of this fraction is finite.

$\frac{2}{9} = 0.22222......$ Therefore the decimal expansion of this fraction recurs.

$\frac{4}{7} = 0.5714285714...$ Therefore the decimal expansion of this fraction recurs.

$\frac{-11}{5} = -2.2$. Therefore the decimal expansion of this fraction is finite.

2 **a** $a = 0.\dot{5}$
$a = 0.5555....$
$10a = 5.5555....$
$10a - a = 5$
$9a = 5$
$a = \frac{5}{9}$

b $b = 1.\dot{8}$
$b = 1.8888$
$10b = 18.8888....$
$10b - b = 17$
$b = \frac{17}{9}$

c $\frac{5}{9} + \frac{17}{9} = \frac{22}{9}$

3 **a** It could be 0.8; 0.5; 2.1; 3.08; etc

b It could be $0.1\dot{2}; 0.\dot{5}; 1.2\dot{4}; 1.0\dot{2}$; etc

c It could be $3.457\dot{8}; 0.000\dot{2}; 1.002\dot{3}$

Exercise 1D

1 either work out the arithmetic mean of these numbers as shown in the book or look for their decimal expansion.

The numbers are 2 and $\frac{9}{4}$
Therefore 2 and 2.25
Numbers in between may be for example
2.1; 2.2; 2.23

2 **a** $\sqrt{2(y-x)}$ when $y = 3$ and $x = -\frac{1}{8}$

$\sqrt{2\left(3-(-\frac{1}{8})\right)} = \frac{5}{2}$ (or 2.5)

b $\frac{5}{2}$ is a rational number

3 **a** The numbers are $\frac{9}{5}$ and $\frac{11}{6}$

Therefore 1.8 and $1.8\dot{3}$
Numbers in between may be for example
1.81;1.82; 1.83.

b **i** The numbers are $-\frac{28}{13}$ and -2
Therefore -2.15384 ... and -2
Numbers in between may be for example
-2.14; -2.12; -2.1

ii infinite

Exercise 1E

1 **a** It is a right angled triangle.
$h^2 = 2^2 + 1.5^2$
$h^2 = 6.25$
$h = 2.5$ cm

b h is rational.

2 **a** $r = \frac{10}{2} = 5$ cm
$A = \pi \times 5^2$
$A = 25\pi$ cm²

b A is irrational.

Exercise 1F

1 **a** **i** $0.5 < \frac{x}{2} \le 1.5$
multiply by 2
$2 \times 0.5 < 2 \times \frac{x}{2} \le 2 \times 1.5$
$1 < x \le 3$

ii make x the subject of the inequality
$3 - x \ge 1$
$3 \ge 1 + x$
$2 \ge x$

b **i**

ii 

c **i** $q = 1.5$ is solution as $1 < 1.5 \le 3$.
$t = \sqrt{5}$ is solution as $1 < \sqrt{5} \le 3$.

ii $q = 1.5$ is solution as $2 \ge 1.5$.
$t = \sqrt{5}$ is not solution as the inequality $2 \ge \sqrt{5}$ is not true.

2 **a** **i** $2x + 1 > -1$
$x > \frac{-2}{2}$
$x > -1$

ii $4 \le x + 1 \le 8$
$4 - 1 \le x + 1 - 1 \le 8 - 1$
$3 \le x \le 7$

iii $2 - x > -1$
$3 > x$

b **i** 

ii

iii 

c substitute each of these numbers in the inequalities

Inequality / p	$2x + 1 > -1$	$4 \le x + 1 \le 8$	$2 - x > -1$
$-\frac{2}{3}$	√		√
$\sqrt{10}$	√	√	
2π	√	√	

Exercise 1G

1 **i** 358.4 = 358 to the nearest unit

ii 24.5 = 25 to the nearest unit

iii 108.9 = 109 to the nearest unit

iv 10016.01 = 10016 to the nearest unit

2 **i** 246.25 = 250 correct to the nearest 10

ii 109 = 110 correct to the nearest 10

iii 1015.03 = 1020 correct to the nearest 10

iv 269 = 270 correct to the nearest 10

3 **i** 140 = 100 correct to the nearest 100.

ii 150 = 200 correct to the nearest 100.

iii 1240 = 1200 correct to the nearest 100.

iv 3062 = 3100 correct to the nearest 100.

4 **i** 105 607 = 106 000 correct to the nearest 1000.

ii 1500 = 2000 correct to the nearest 1000.

iii 9640 = 10 000 correct to the nearest 1000.

iv 952 = 1000 correct to the nearest 1000.

5 Any x where $150 \le x < 250$

6 Any x where $2500 \le x < 3500$

Any x where $5.5 \le x < 6.5$

Exercise 1H

1 **i** 45.67 = 45.7 correct to 1 d.p.

ii 301.065 = 301.1 correct to 1 d.p.

iii 2.401 = 2.4 correct to 1 d.p.

iv 0.09 = 0.1 correct to 1 d.p.

2 **i** 0.0047 = 0.00 correct to 2 d.p.

ii 201.305 = 201.31 correct to 2 d.p.

iii 9.6201 = 9.62 correct to 2 d.p.

iv 28.0751 = 28.08 correct to 2 d.p.

3 **i** 10.0485 = 10.049 correct to the nearest thousandth.

ii 3.9002 = 3.900 correct to the nearest thousandth.

iii 201.7805 = 201.781 correct to the nearest thousandth.

iv 0.008 41 = 0.008 correct to the nearest thousandth.

4 $\dfrac{\sqrt{1.8}}{3.04 \times 0.012^2} = 3064.786153.$

i 3064.8 (1 d.p.)

ii 3064.79 (2 d.p.)

iii 3064.786 (3 d.p.)

iv 3100 correct to the nearest 100

v 3000 correct to the nearest 1000

5 $\dfrac{(p+q)^3}{p+q} = 15.6025$

i 15.60 (2 d.p.)

ii 15.603 (3 d.p.)

iii 16 correct to the nearest unit

iv 20 correct to the nearest 10

6 Any x where $2.365 \le x < 2.375$

7 Any x where $4.05 \le x < 4.15$

Exercise 1I

1 **i** 106 has **3** significant figures as all non-zero digits are significant and zeros between non-zero digits are significant.

ii 200 has **1** significant figure as trailing zeros in a whole number are not significant.

iii 0.02 has **1** significant figure as all non-zero digits are significant and zeros to the left of the first non-zero digit are **not** significant.

iv 1290 has **3** significant figures as trailing zeros in a whole number are not significant.

v 1209 has **4** significant figures as all non-zero digits are significant and zeros between non-zero digits are significant.

2 **i** 280 = 300 (1 s.f.)

ii 0.072 = 0.07 (1 s.f.)

iii 390.8 = 400 (1 s.f.)

iv 0.001 32 = 0.001 (1 s.f.)

3 **i** 355 = 360 (2 s.f.)

ii 0.0801 = 0.080 (2 s.f.)

iii 1.075 = 1.1(2 s.f.)

iv 1560.03 = 1600 (2 s.f.)

4 **i** 2971 = 2970 (3 s.f.)

ii 0.3259 = 0.326 (3 s.f.)

iii 10410 = 10400 (3 s.f.)

iv 0.5006 = 0.501(3 s.f.)

5 $\dfrac{\sqrt{8.7 + 2 \times 1.6}}{0.3^4} = 425.881\,192\,9$

a 400 correct to 1 significant figures

b 426 correct to 3 significant figures

c 425.9 correct to 1 decimal place

d 425.88 correct to the nearest hundredth

6 $\pi = 3.141592654$

a 3 correct to the nearest unit

b 3.14 correct to 2 d.p.

c 3.1 correct to 2 s.f.

d 3.142 correct to 3 d.p.

7 **a** $238 = 200$ (1 s.f.)

b $4609 = 4610$ (3 s.f.)

c $2.7002 = 2.70$ (3 s.f.)

8 **a** $\dfrac{\sqrt[3]{3.375}}{1.5^2 + 1.8} = 0.3703703704$

b **i** 0.37 **ii** 0.370 **iii** 0.3704

Exercise 1J

1 **a** $A = \pi r^2$

$10.5 = \pi r^2$

$\dfrac{10.5}{\pi} = r^2$

$r = \sqrt{\dfrac{10.5}{\pi}}$

$r = 1.828$ cm (4 s.f.)

b $C = 2\pi r$

$C = 2\pi \times \sqrt{\dfrac{10.5}{\pi}}$

$C = 11$ cm (2 s.f.)

2 **a** $\dfrac{\sqrt{2} + \sqrt{10}}{2} = 2.288$ (4 s.f.)

b substitute the values of p and q in the formula.

$(p+q)^2 = \left(\sqrt{2} + \sqrt{10}\right)^2 = 20.9$ (3 s.f.)

c $\sqrt{2} \times \sqrt{10} = 4.5\ \text{cm}^2$ (2 s.f.)

Exercise 1K

1 **a** $298 \times 10.75 \approx 300 \times 10 = 3000$

b $3.8^2 \approx 3.8 \times 3.8 = 4 \times 4 = 16$

c $\dfrac{147}{11.02} \approx \dfrac{150}{10} = 15$

d $\sqrt{103} \approx \sqrt{100} = 10$

2 $210 \times 18 \approx 200 \times 20 = 4000$ pipes.

3 population density $= \dfrac{\text{total population}}{\text{land area}}$

population density $= \dfrac{127\,076\,183}{377\,835}$

population density $\approx \dfrac{120\,000\,000}{400000}$

population density ≈ 300 people per km^2

4 Number of reams $= \dfrac{9000}{500}$

Number of reams $\approx \dfrac{10000}{500}$

Number of reams ≈ 20

5 Average speed $= \dfrac{\text{distance travelled}}{\text{time taken}}$

Average speed $= \dfrac{33}{1.8}$

Average speed $\approx \dfrac{30}{2}$

Average speed $\approx 15\ \text{km h}^{-1}$

6 Number of visitors per year $= 53000 \times 365$

Number of visitors per year $\approx 50000 \times 400$

Number of visitors per year ≈ 20000000

7 estimate the area of the square using reasonable numbers.

Area of square $= 100.1 \times 100.1$

Area of square $= 100 \times 100$

Area of square $= 10\,000\ \text{m}^2$

Therefore Peter's calculation is not correct. 10 000 is far bigger than 1020.01

Exercise 1L

1 **a** substitute the values of a and of b in the given formula.

$3a + b^3 = 3 \times 5.2 + 4.7^3 = 119.423$

b Percentage error $= \left|\dfrac{v_A - v_E}{v_E}\right| \times 100\%$

Percentage error $= \left|\dfrac{140 - 119.423}{119.423}\right| \times 100\%$

Percentage error $= 17.2\%$ (3 s.f.)

2 **a** Actual final grade $= \dfrac{8.3 + 6.8 + 9.4}{3}$

Actual final grade $= 8.17$ (3 s.f.)

b The three grades rounded are 8, 7 and 9.

Approximate final grade $= \dfrac{8 + 7 + 9}{3}$

Approximate final grade $= 8$

c Percentage error $= \left|\dfrac{8 - 8.1666}{8.1666}\right| \times 100\%$

Percentage error $= 2.04\%$ (3 s.f.)

3 **a** Exact area $= 5.34 \times 3.48$

Exact area $= 18.5832\ \text{m}^2$

b Length $= 5.3$ m

Width $= 3.5$ m

c Approximate area $= 18.55\ \text{m}^2$

Percentage error $= \left|\dfrac{18.55 - 18.5832}{18.5832}\right| \times 100\%$

Percentage error $= 0.179\%$ (3 s.f.)

4 **a** $A = \pi r^2$

$89 = \pi r^2$

$r = \sqrt{\dfrac{89}{\pi}}$ cm

$r = 5.323$ m (3 d.p.)

b $C = 2\pi r$

$C = 2\pi \sqrt{\dfrac{89}{\pi}}$

$C = 33.4$ m (3 s.f.)

c Approximate value for perimeter = 30 m
Accepted value for perimeter = 33.4 m

Percentage error = $\left|\frac{30-33.4}{33.4}\right| \times 100\%$

Percentage error = 10% (2 s.f.)

Exercise 1M

1 2.5×10^{-3}; 10^{10}

2 a number is written in standard form if it is written as $a \times 10^k$ where $1 \le a < 10$ and k is an integer.

a $135\,600 = 1.356 \times 10^5$ or 1.36×10^5 (3 s.f.)
b $0.00245 = 2.45 \times 10^{-3}$
c $16\,000\,000\,000 = 1.6 \times 10^{10}$
d $0.000\,108 = 1.08 \times 10^{-4}$
e $0.23 \times 10^3 = 2.3 \times 10^2$

3 First, write each number in standard form 2.3×10^6
3.4×10^5
$0.21 \times 10^7 = 2.1 \times 10^6$
$215 \times 10^4 = 2.15 \times 10^6$
Now write in order 3.4×10^5; $0.21 \times 10^7 = 2.1 \times 10^6$; $215 \times 10^4 = 2.15 \times 10^6$; 2.3×10^6

4 3.621×10^4
$31.62 \times 10^2 = 3.162 \times 10^3$
$0.3621 \times 10^4 = 3.621 \times 10^3$
0.3621×10^3
3.621×10^4; $0.3621 \times 10^4 = 3.621 \times 10^3$; $31.62 \times 10^2 = 3.162 \times 10^3$.

Exercise 1N

1 **a** $x \times y = 6.3 \times 10^6 \times 2.8 \times 10^{10} = 1.764 \times 10^{17}$ or 1.76×10^{17} (3 s.f.)

b $\frac{x}{y} = \frac{6.3 \times 10^6}{2.8 \times 10^{10}} = 2.25 \times 10^{-4}$

c $\sqrt{\frac{x}{y}} = \sqrt{\frac{6.3 \times 10^6}{2.8 \times 10^{10}}} = 1.5 \times 10^{-2}$

2 **a** the arithmetic mean between a and b is simply $\frac{a+b}{2}$.

Arithmetic mean = $\frac{2.5 \times 10^6 + 3.48 \times 10^6}{2}$
Arithmetic mean = 2 990 000
Arithmetic mean = 2.99×10^6

b nearest million is the nearest multiple of 1 000 000

2 990 000 = 3 000 000 correct to the nearest million or 3×10^6

3 **a** $t = 22.05 \times 10^8$
$t = 2.205 \times 10^9$

b $\frac{t}{q} = \frac{22.05 \times 10^8}{3.15 \times 10^6} = 700$

c 7×10^2

4 **a** $x = 225 \times 10^8$
$x = 2.25 \times 10^{10}$

b $x^2 = (225 \times 10^8)^2$
$x = 5.0625 \times 10^{20}$
$x^2 > 10^{20}$ because both have the same exponent for 10 when written in standard form and $5.0625 > 1$ therefore the statement is true.

c **i** substitute the value of x in the given expression.

$\frac{x}{\sqrt{x}} = \frac{225 \times 10^8}{\sqrt{225 \times 10^8}} = 150\,000$

ii Write your answer in standard form
$150\,000 = 1.5 \times 10^5$

Exercise 1O

1 **a** km h^{-2} or km/h^2
b kg m^{-3} or kg/m^3
c m s^{-1} or m/s

2 **a** **i** decagrams **ii** centisecond
iii millimetre **iv** decimetre

3 **a** 32 km = 32×10^3 m = 32 000 m
b 0.87 m = 0.87×10^{-1} dam = 0.087 dam
c 128 cm =
128×10^{-2} m = 1.28 m

4 **a** 500 g = 500×10^{-3} = 0.5 kg
b 357 kg = 357×10^2 dag = 35 700 dag
c 1080 dg = 1080×10^3 hg = 1.08 hg

5 **a** 0.080 s = 0.080×10^3 = 80 ms
b 1200 s = 1200×10^{-1} das = 120 das
c 0.8 hs = 0.8×10^3 ds = 800

6 **a** 67 800 000 mg = 67 800 000 $\times 10^{-6}$ = 67.8 kg = 68 kg correct to the nearest kg.
b 35 802 m = 35 802 $\times 10^{-3}$ km = 35.802 km = 36 km correct to the nearest km
c 0.654 g = 0.654×10^3 mg = 6.54×10^2 mg

Exercise 1P

1 **a** 2.36 m^2 = 2.36×10^4 cm^2 = 23 600 cm^2
b 1.5 dm^2 = 1.5×10^{-4} dam^2 = 0.000 15 dam^2
c 5400 mm^2 = 5400×10^{-2} cm^2 = 54 cm^2
d 0.06 m^2 = 0.06×10^6 mm^2 = 60 000 mm^2
e 0.8 km^2 = 0.8×10^2 hm^2 = 80 hm^2
f 35 000 m^2 = 35 000 $\times 10^{-6}$ km^2 = 0.035 km^2

2 **a** 5 m^3 = 5×10^6 cm^3 = 5 000 000 m^3
b 0.1 dam^3 = 0.1×10^3 m^3 = 1×10^2 m^3
c 3 500 000 mm^3 = 3 500 000 $\times 10^{-6}$ dm^3
= 3.5×10^0 dm^3
d 255 m^3 = 255×10^9 mm^3 = 2.55×10^{11} mm^3
e 12 000 m^3 = 12000 $\times 10^{-3}$ dam^3
= 1.2×10^1 dam^3

f $0.7802\ \text{hm}^3 = 0.7802 \times 10^3\ \text{dam}^3$
$= 7.802 \times 10^2\ \text{dam}^3$
$= 7.80 \times 10^2\ \text{dam}^3$ (3 s.f.)

3 the area of a square with side length l is l^2.

a Area = $l \times l$
Area = 13^2
Area = 169 cm^2

b $169\ \text{cm}^2 = 169 \times 10^{-4}\ \text{m}^2 = 0.0169\ \text{m}^2$

4 the volume of a cube with side length (or edge) l is l^3.

a Volume = l^3
Volume = 0.85^3
Volume = 0.614125 m^3 or 0.614 m^3 (3 s.f.)

b $0.614125\ \text{m}^3 = 0.614125 \times 10^6\ \text{cm}^3$
$= 614125\ \text{m}^3$ or $614000\ \text{cm}^3$ (3 s.f.)

5 convert all the measurements to the same unit.
$0.081\ \text{dam}^2 = 8.1\ \text{m}^2$;
$8000000\ \text{mm}^2 = 8\ \text{m}^2$;
$82\ \text{dm}^2 = 0.82\ \text{m}^2$
$7560\ \text{cm}^2 = 0.756\ \text{m}^2$
$0.8\ \text{m}^2$
Therefore the list from smallest is
$7560\ \text{cm}^2$; $0.8\ \text{m}^2$; $82\ \text{dm}^2$ $8000000\ \text{mm}^2$;
$0.081\ \text{dam}^2$

6 convert all the measurements to the same unit.
$11.2\ \text{m}^3$;
$1200\ \text{dm}^3 = 1.2\ \text{m}^3$
$0.01\ \text{dam}^3 = 10\ \text{m}^3$
$11020000000\ \text{mm}^3 = 11.02\ \text{m}^3$
$10900000\ \text{cm}^3 = 10.9\ \text{m}^3$
Therefore the list from smallest is
$1200\ \text{dm}^3$; $0.01\ \text{dam}^3$; $10\,900\,000\ \text{cm}^3$;
$11020000000\ \text{mm}^3$; $11.2\ \text{m}^3$

Exercise 1Q

1 a change all to seconds
1 d = 24 h = 24 × 60 min
= 24 × 60 × 60 s = 86 400 s
2 h = 2 × 60 min = 2 × 60 × 60 s = 7200 s
23 min = 23 × 60 s = 1380 s
Therefore
1 d 2 h 23 m = 86 400 s + 7200 s + 1380 s
= 94 980 s

b 94 980 s = 95 000 (nearest 100)

2 a change all to seconds
2 d = 48 h = 48 × 60 min = 48 × 60 × 60 s
= 172 800 s
5 min = 5 × 60 s = 300 s
Therefore
2 d 5 min = 172 800 s + 300 s = 173 100 s

b 173 100 s = 1.731×10^5 s or 1.73×10^5 s (3 s.f.)

3 a 5 l = 5 × 10^3 ml = 5000 ml

b 0.56 ml = 0.56 × 10^{-5} hl = 0.0000056 hl

c 4500 dal = 4500 × 10^3 cl = 4 500 000 cl

4 1 l = 1 dm^3

a 500 l = 500 dm^3 = 500 × 10^3 cm^3 = 5 × 10^5 cm^3

b 145.8 dl = 14.58 l = 1.458 × 10^1 dm^3
or 1.46 × 10^1 dm^3 (3 s.f.)

c 8 hl = 800 l = 800 dm^3 = 800 × 1000 cm^3
= 8 × 10^3 cm^3

5 a 12.5 dm^3 = 12.5 l = 13 correct to the nearest unit.

b 0.368 m^3 = 0.368 × 10^3 dm^3 = 368 dm^3
= 368 l = 3.68 hl
= 4 hl correct to the nearest unit.

c 809 cm^3 = 809 × 10^{-3} dm^3 = 0.809 dm^3
= 0.809 l = 80.9 cl
= 81 cl correct to the nearest unit.

6 Average speed = $\dfrac{\text{distance travelled}}{\text{time taken}}$

a Average speed = $\dfrac{\text{distance travelled}}{\text{time taken}}$

$40\ \text{m min}^{-1} = \dfrac{3000\ \text{m}}{\text{time taken}}$

time taken = $\dfrac{3000\ \text{m}}{40\ \text{m min}^{-1}}$

time taken = 75 min

b 75 min = 75 × 60 min = 4500 s

7 volume of a cube = l^3

a Volume = 1.5^3 = 3.375 m^3

b 3.375 m^3 = 3.375 × 10^3 dm^3 = 3375 dm^3

c 3375 dm^3 = 3375 l and 3375 l < 4000 l
therefore 4000 l of water cannot be poured in this container. Only 3375 l can be poured.

8 a $\frac{4}{5}$ of 220 cm^3 = 176 cm^3
176 cm^3 = 176 × 10^{-3} dm^3 = 0.176 1

b $\dfrac{1.5}{0.176}$ = 8.52 tea cups therefore Mercedes can serve up to 8 tea cups.

9 a Average speed = $\dfrac{\text{distance travelled}}{\text{time taken}}$

$800\ \text{km h}^{-1} = \dfrac{6900\ \text{km}}{\text{time taken}}$

time taken = $\dfrac{6900\ \text{km}}{800\ \text{km h}^{-1}}$

time taken = 8.625 h or 8.63 h (3 s.f.)

b Average speed = $\dfrac{\text{distance travelled}}{\text{time taken}}$

Average speed = $\dfrac{1393\text{km}}{2\text{h}}$

Average speed = 696.5 km h^{-1} or 697 km h^{-1}

c Time travelling = 8.625 h + 2 h + 1.5 h
= 12.125 h
Arrival time = 10 + 12.125 = 22.125 h or 10:13 PM

Exercise 1R

1 **a** $t_c = t_k - 273.15$
$t_c = 280 - 273.15$
$t_c = 280 - 273.15 = 6.85$
6.85 °C = 6.9 °C correct to one tenth of degree

b $80 = \frac{9}{5} \times t_C + 32$
$t_C = (80 - 32)\frac{5}{9}$
$t_C = \frac{80}{3} = 26.\dot{6}$
$26.\dot{6}$ °C = 26.7 °C correct to one tenth of degree

2 **a** $t_F = \frac{9}{5} \times 21 + 32$
$t_F = \frac{349}{5} = 69.8$
69.8 °F = 70 °F correct to the nearest degree.

b $t_F = \frac{9}{5} \times 2 + 32$
$t_F = \frac{178}{5} = 35.6$
35.6 °F = 36 °F correct to the nearest degree.

3 **a** $t_c = 290 - 273.15 = 16.85$
Therefore 290 K = 16.85 °C or 16.9 °C (3 s.f.)

b "hence" means use the preceding answer to solve this part question.

290 K = 16.85 °C
Also $t_F = \frac{9}{5} \times 16.85 + 32$
$t_F = \frac{9}{5} \times 16.85 + 32 = 62.33$
Therefore 290 K = 62.33 °F or 62.3 °F (3 s.f.)

4 **a** make t_K the subject of the formula.
$t_C = t_K - 273.15$
$t_K = t_C + 273.15$

b make t_C the subject of the formula
$t_F = \frac{9}{5} \times t_C + 32$
$t_C = \frac{5}{9}(t_F - 32)$

Review exercise

Paper 1 style questions

1

	5	$\frac{\pi}{2}$	−3	$\frac{5}{4}$	2.3
$\mathbb{N}$	√				
$\mathbb{Z}$	√		√		
$\mathbb{Q}$	√		√	√	√
$\mathbb{R}$	√	√	√	√	√

2 **a** $\sqrt{2}$ **b** $\sqrt{2} = 1.4142$

c $14.1 \times 10^{-1} = 1.41 \times 10^0$
14.1×10^2
$\sqrt{2} \approx 1.4142 \times 10^0$
$0.00139 \times 10^2 = 1.39 \times 10^{-3}$
$1414 \times 10^{-2} = 1.414 \times 10^1$
0.00139×10^2; 14.1×10^{-1}; $\sqrt{2}$; 1414×10^{-2}; 1.4×10^2

3 **a** 2690 kg = 2.69×10^3 kg

b **i** 2.7×10^3 kg = 2700 kg

ii percentage error formula
Percentage error = $\left|\frac{v_A - v_E}{v_E}\right| \times 100\%$

$$\text{Percentage error} = \left|\frac{v_A - v_E}{v_E}\right| \times 100\%$$
$$\text{Percentage error} = \left|\frac{2700 - 2690}{2690}\right| \times 100\%$$
Percentage error = 0.372% (3 s.f.)

4 **a** 299 792 458 m s^{-1} = 300 000 000 m s^{-1}

b m s^{-1} means metres per second therefore the answer from **a** gives you the distance traveled in 1 second.
$1\text{ s} \xrightarrow{travels} 300\,000\,000\text{ m}$
300 000 000 m = 300 000 000 × 10^{-3} km
= 300 000 km

c $1\text{ s} \xrightarrow{travels} 300\,000\text{ km}$
$3600\text{ s} \xrightarrow{travels} 300\,000\text{ km} \times 3600$
= 1 080 000 000 km
1 080 000 000 km = 1.08×10^9 km
Therefore the average velocity is 1.08×10^9 km h^{-1}

5 **a** Exact weight of one book = $\frac{52200}{90} = 580$ g
580 g = 580 × 10^{-3} kg = 0.580 kg

b 0.580 kg = 0.6 kg (1 s.f.)

c Accepted value = 0.6 kg
Estimated value = 0.4 kg
$$\text{Percentage error} = \left|\frac{v_A - v_E}{v_E}\right| \times 100\%$$
$$\text{Percentage error} = \left|\frac{0.4 - 0.6}{0.6}\right| \times 100\%$$
Percentage error = 33.3% (3 s.f.)

6 **a** 1560 cm^3 = 1560 × 10^{-3} dm^3 = 1.56 dm^3

b 1.56 dm^3 = 1.56 l
$\frac{3}{4}$ of 1.56 l = 1.17 l

c **i** $\frac{25}{1.17} \approx 21.4$ jars
Therefore Sean pours 21 jars.

ii 21 × 1.17 = 24.571
25 − 24.57 = 0.431

7 **a** $x = \frac{30y^2}{\sqrt{y+1}}$ when $y = 1.25$

$x = \frac{30(1.25)^2}{\sqrt{1.25+1}}$

$x = 31.25$

b $31.25 = 31.3$ (3 s.f.)

c $31.3 = 3.13 \times 10^1$

8 **a** $A = x^2$

b **i** $2.56\ \text{km}^2 = 2.56 \times 10^6\ \text{m}^2 = 2\,560\,000\ \text{m}^2$

$x^2 = 2\,560\,000$

$x = \sqrt{2\,560\,000}$

$x = 1600$ m

ii Perimeter = 1600×4

Perimeter = 6400 m

9 **a** $t_F = \frac{9}{5} \times t_K - 459.67$

$t_F = \frac{9}{5} \times 300 - 459.67$

$t_F = 80.33$ or 80.3 (3 s.f.)

b $t_K = \frac{9}{5} \times t_K - 459.67$

$100 = \frac{9}{5} \times t_K - 459.67$

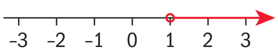
$t_K = \frac{5}{9}(100 + 459.67)$

$t_F = 310.927.... = 311$ correct to the nearest unit

10 **a** $2x + 5 > x + 6$

$x > 1$

b



c $1 = 1$

$\frac{\pi}{4} = 0.785.... < 1$

$-5 < 1$

$\sqrt{3} = 1.732.... > 1$ ✓

$2.0\dot{6} = 2.06666\ldots > 1$ ✓

$\frac{101}{100} = 1.01 > 1$ ✓

$1.2 \times 10^{-3} = 0.0012 < 1$

Therefore

$\sqrt{3};\ 2.0\dot{6};\ \frac{101}{100}$

11 **a** Area = 210 mm × 297 mm

Area = 62370 mm^2

b $62370\ \text{mm}^2 = 62370 \times 10^{-6}\ \text{m}^2 = 0.062370\ \text{m}^2$

c $1\ \text{m}^2 \xrightarrow{\text{weighs}} 75$ g

$0.062370\ \text{m}^2 \xrightarrow{\text{one page weighs}} 0.062370 \times 75$

$= 4.67775$ g $= 4.68$ g (3 s.f.)

d $4.68 \times 500 = 2340$ g

$2340\ \text{g} = 2340 \times 10^{-3}\ \text{kg} = 2.34$ kg

Review exercise

Paper 2 style questions

1 **a** Perimeter of the field = $2 \times 2500 + 2 \times 1260$

Perimeter of the field = 7520 m

$7520\ \text{m} = 7520 \times 10^{-3}\ \text{km} = 7.52$ km

b Cost of fencing the field = 7.52×327.64

Cost of fencing the field = 2463.85 (2 d.p.)

$V_A = 7.6 \times 327.64 = 2490.064$

c Percentage error = $\left|\frac{v_A - v_E}{v_E}\right| \times 100\%$

Percentage error = $\left|\frac{2490.064 - 2463.85}{2463.85}\right| \times 100\%$

Percentage error = 1.06% (3 s.f.)

d Area of the field = 2500×1260

Area of the field = $3\,150\,000\ \text{m}^2$

Area of the field = $3\,150\,000 \times 10^{-6}\ \text{km}^2$

$= 3.15\ \text{km}^2$

2 **a** Radius of semicircles = $\frac{400}{2} = 200$ m

Length of circumference = $2\pi r$

Length of circumference = $2\pi \times 200 = 400\pi$

Perimeter = $2 \times 800 + 400\pi$

Perimeter = 2856.637... m

= 2857 m correct to the nearest metre.

b Number of laps that Elger runs

$= \frac{\text{total distance run by Elger}}{\text{perimeter of running track}}$

Number of laps that Elger runs

$= \frac{14200}{2856.637...}$

Number of laps that Elger runs = 4.97

Therefore Elger runs 4 complete laps around the track.

c convert the distance to km

$2856.637\ldots\ \text{m} = 2856.637\ldots \times 10^{-3}\ \text{km}$

$= 2.856637\ldots$ km

average speed = $\frac{\text{distance travelled}}{\text{time taken}}$

$19\ \text{km h}^{-1} = \frac{2.856637\ldots\ \text{km}}{\text{time taken}}$

time taken = $\frac{2.856637\ldots\ \text{km}}{19\ \text{km h}^{-1}}$

time taken = 0.150 h (3 s.f.)

d average speed = $19\ \text{km h}^{-1} = \frac{19\ \text{km}}{1\ \text{h}} = \frac{19000\ \text{m}}{60\ \text{min}}$

$= \left(\frac{19000}{60}\right) \text{m min}^{-1}$

$\left(\frac{19000}{60}\right) \text{m min}^{-1} = \frac{14\,200\ \text{m}}{\text{time taken}}$

time taken = $\frac{14\,200\ \text{m}}{\left(\frac{19000}{60}\right) \text{m min}^{-1}}$

time taken = 44.842 min (5 s.f.)

e Percentage error = $\left|\frac{v_A - v_E}{v_E}\right| \times 100\%$

Percentage error = $\left|\frac{44 - 44.842}{44.842}\right| \times 100\%$

Percentage error = 1.88% (3 s.f.)

3 **a** Diameter = 2.5 cm

Radius = $\frac{2.5}{2}$ = 1.25 cm

Volume of one chocolate = $\frac{4}{3}\pi r^3$

Volume of one chocolate = $\frac{4}{3}\pi(1.25)^3$

Volume of one chocolate = 8.18123.... cm^3

= 8.18 cm^3 (2 d.p.)

b first convert the measurements to cm.

Radius of cylindrical box = 12.5 mm

= 1.25 cm

Volume of cylindrical box = $\pi r^2 h$

Volume of cylindrical box = $\pi\,(1.25)^2 15$

Volume of cylindrical box = 73.63107... cm^3

= 73.63 cm^3(2 d.p.)

c Number of chocolates in the box = $\frac{15}{2.5} = 6$ chocolates

d Volume occupied by the chocolates

= 8.18123.... × 6 = 49.087 .... cm^3

Volume **not** occupied by the chocolates

= volume of box – volume occupied by chocolates

Volume **not** occupied by the chocolates

= 73.63107... – 49.087.... = 24.5 cm^3 (3 s.f.)

e 24.5 cm^3 = 24.5 × 10^3 mm^3 = 24 500 mm^3

f 2.45 × 10^4 mm^3

2 Descriptive statistics

Answers

Exercise 2A

1 a Discrete b Continuous
c Discrete d Discrete
e Continuous f Discrete
g Continuous h Continuous
i Continuous j Discrete
k Continuous l Discrete

2 a Biased b Random
c Biased d Random
e Biased

Exercise 2B

1

Number of goals	Frequency
0	4
1	7
2	7
3	4
4	1
5	2

2

Number of heads	Frequency
0	1
1	1
2	4
3	4
4	3
5	7
6	9
7	4
8	5
9	2
10	4
11	3
12	3

3

Age	Frequency
9	4
10	9
11	8
12	7
13	4
14	1
15	4
16	3

4

Number of crisps	Frequency
88	3
89	6
90	16
91	3
92	2

5

Number	Frequency
1	7
2	9
3	11
4	6
5	7
6	10

6 $m = 6$, $n = 3$

Exercise 2C

1 Answers will depend on width of class intervals chosen. Example:

a

Number	Frequency
$0 \le x < 5$	1
$5 \le x < 10$	7
$10 \le x < 15$	3
$15 \le x < 20$	4
$20 \le x < 25$	6
$25 \le x < 30$	1
$30 \le x < 35$	5
$35 \le x < 40$	0
$40 \le x < 45$	2
$45 \le x < 50$	1

b

Number	Frequency
$10 \le x < 20$	7
$20 \le x < 30$	5
$30 \le x < 40$	7
$40 \le x < 50$	5
$50 \le x < 60$	7
$60 \le x < 70$	5
$70 \le x < 80$	5
$80 \le x < 90$	2
$90 \le x < 100$	2

c

Number	Frequency
$1 \le x < 3$	3
$3 \le x < 5$	7
$5 \le x < 7$	4
$7 \le x < 9$	3
$9 \le x < 11$	6
$11 \le x < 13$	3
$13 \le x < 15$	4
$15 \le x < 17$	3
$17 \le x < 19$	1
$19 \le x < 21$	1

Exercise 2D

1 a

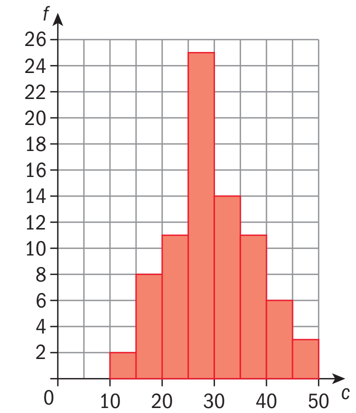
Class	Lower boundary	Upper boundary
9–12	8.5	12.5
13–16	12.5	16.5
17–20	16.5	20.5
21–24	20.5	24.5

b

Time (t seconds)	Lower boundary	Upper boundary
$2.0 \le t < 2.2$	2.0	2.2
$2.2 \le t < 2.4$	2.2	2.4
$2.4 \le t < 2.6$	2.4	2.6

Exercise 2E

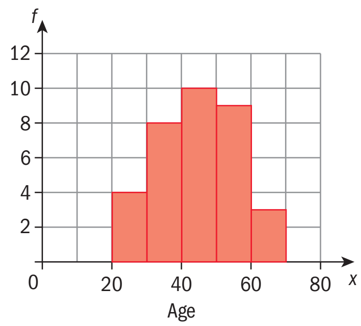
1



2 a Lower boundaries are 20, 30, 40, 50, 60

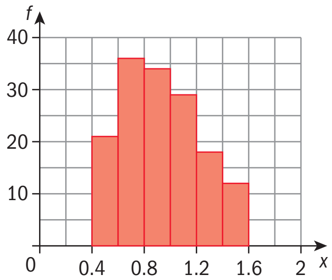
Upper boundaries are 30, 40, 50, 60, 70

b



3 a Lower boundary of the third class is 0.8 and the upper boundary is 1.0

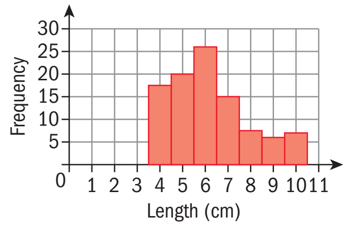
b



4 a Lower boundaries are 3.5, 4.5, 5.5, 6.5, 7.5, 8.5, 9.5

Upper boundaries are 4.5, 5.5, 6.5, 7.5, 8.5, 9.5, 10.5

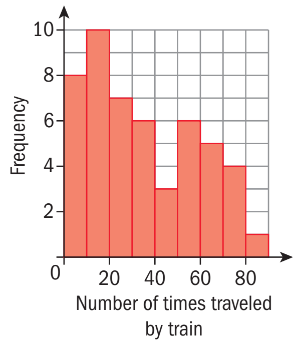
b



5 a

Number	Frequency
$0 \le x < 10$	8
$10 \le x < 20$	10
$20 \le x < 30$	7
$30 \le x < 40$	6
$40 \le x < 50$	3
$50 \le x < 60$	6
$60 \le x < 70$	5
$70 \le x < 80$	4
$80 \le x < 90$	1

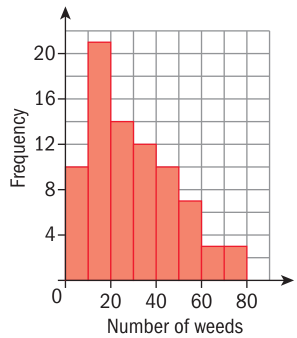
b



6 a

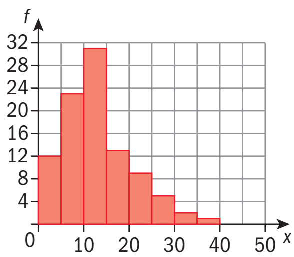
Number of weeds	frequency
$0 \le x < 10$	10
$10 \le x < 20$	21
$20 \le x < 30$	14
$30 \le x < 40$	12
$40 \le x < 50$	10
$50 \le x < 60$	7
$60 \le x < 70$	3
$70 \le x < 80$	3

b

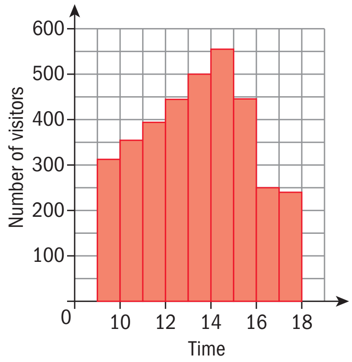


7 a The lower boundary of the fourth group is 15.5 and the upper boundary is 20.5

b



8



Exercise 2F

1 **a** Arrange in order: 1 1 3 7 8 9 10

Mode = 1

Median = 4th entry = 7

Mean = $\frac{1+1+3+7+8+9+10}{7} = \frac{39}{7}$ = 5.57 (3 sf)

b Arrange in order: 2 3 3 4 5 5 5 6 6 8 11 13

Mode = 5

Median = 6.5th entry = 5

Mean = $\frac{2+3+3+4+5+5+5+6+6+8+11+13}{12}$

$= \frac{71}{12}$ = 5.92 (3 sf)

2 **a** Arrange in order: 1.52 1.52 1.67 1.74 1.83 1.91

Median = 3.5th entry = $\frac{1.67+1.74}{2} = \frac{3.41}{2}$

= 7.71(3 sf)

b Mode = 1.52

c Mean = $\frac{21+34+17+22+56+38}{6}$ = 31.3

d Arrange in order: 48.6 48.6 54.7 55.1 63.2 77.9

Median = 3.5th entry = $\frac{54.7+55.1}{2}$ = 54.9

e Mean = $\frac{48.6+48.6+54.7+55.1+63.2+77.9}{6}$ = 58.0

3 **a** Arrange in order: 8.9 12.6 18.7 22.6 26.3 31.8 33.5 45.3

Median = 4.5th entry = $\frac{22.6+26.3}{3}$ = 24.45

b Mean = $\frac{26.3+12.6+33.5+8.9+18.7+22.6+31.8+45.3}{8}$

= 25.0

4 If the mode is 5 then s = 5 because we need more 5s than other numbers.

If the mean is 6.5 then

$$\frac{(1+1+2+3+5+5+5+7+8+9+10+t+12+12)}{14} = 6.5$$

80 + t = 6.5 × 14 = 91

So, t = 11

5 **a** $\frac{(76+54+65)}{3} = 65$

b $\frac{(195+x)}{4} = 68$

So, 195 + x = 68(4) = 272 x = 77

6 **a** Zoe's total = 5 × 81 = 405

Shun's total = 78 × 3 = 234

405 + x = 80(6) = 480 x = 75

b 234 + x = 80(4) = 320 x = 86

Exercise 2G

1 **a** Modal score = 4 (it has the highest frequency)

b Median = $\frac{29+1}{2}$ =15th entry = 4

c Mean = $\frac{1\times4+2\times7+3\times3+4\times8+5\times5+6\times2}{4+7+3+8+5+2}$ = 3.31

2 **a** Number of children = 4 + 3 + 8 + 5 + 4 + 1

= 25

b Highest frequency = 8, therefore modal number of visits = 2

c Mean = $\frac{0\times4+1\times3+2\times8+3\times5+4\times4+5\times1}{4+3+8+5+4+1}$ = 2.2

3 **a** n = 30 – (4 + 5 + 3 + 6 + 5) = 7

b Mean = $\frac{1\times4+2\times5+3\times3+4\times7+5\times6+6\times5}{4+5+3+7+6+5}$ = 3.7

c 4 because it has the highest frequency.

4 **a** Mean = $\frac{1\times1+2\times6+3\times19+4\times34+5\times32+6\times18+7\times10}{1+6+19+34+32+18+10}$

$= \frac{544}{120}$ = 4.53

b $\frac{(34+32)}{120}\times100$ = 55%

c 4 because it has the highest frequency

Exercise 2H

1 **a** 24 ≤ t < 26

b Use GDC. See Chapter 12 for help.

2 **a** 70 ≤ s < 80

b Use GDC. See Chapter 12 for help.

3 **a** 40 ≤ x < 50

b Use GDC. See Chapter 12 for help.

Exercise 2I

1 **a** N = the total number of times = 50

b 6 + a = 14 a = 8

b = 50 – (6 + 8 + 10 + 5 + 7) = 14

c = 24 + 14 = 38

Questions 2–6: All the answers can be read from the graphs.

Exercise 2J

All the answers can be read from the graphs.

Exercise 2K

All the answers can be read from the graphs.

Exercise 2L

1 **a** **i** range = 21 – 2 = 19
IQR (from GDC) = 11 – 2 = 9

b **i** range = 16 – 3 = 13
IQR (from GDC) = 12 – 8 = 4

c **i** range = 25 – 18 = 7
IQR (from GDC) = 23.5 – 19 = 4.5

Exercise 2M

Use GDC. See Chapter 12 in the book for help.

3 Statistical applications

Answers

Skills check

1 **a** $15^2 + h^2 = 25^2$

$h = 20\,\text{cm}$

b $x^2 + x^2 = 10^2$

$2x^2 = 100$

$x^2 = 50$

$x = \sqrt{50}\,\text{cm}$ or 7.07 cm (3 s.f.)

2 **a** **i** Using the midpoint formula.

Let M be the midpoint between A and B.

$M = \left(\frac{-3+3}{2}, \frac{5+7}{2}\right)$

$M = (0, 6)$

ii Let d be the distance between A and B.

$d = \sqrt{(3-(-3))^2 + (7-5)^2}$

$d = \sqrt{40}$ or 6.32 (3 s.f.)

b Using the midpoint formula and set two equations in p and q.

$(2.5, 1) = \left(\frac{2+q}{2}, \frac{p-4}{2}\right)$

$\frac{2+q}{2} = 2.5$ and $\frac{p-4}{2} = 1$

Therefore

$q = 3$ and $p = 6$

Exercise 3A

1 Using the gradient formula.

a $m = \frac{9-7}{0-2}$

$m = -1$

b $m = \frac{-9-7}{0-2}$

$m = 8$

c $m = \frac{9-(-7)}{0-2}$

$m = -8$

d $m = \frac{-9-(-7)}{0-2}$

$m = 1$

2 **a** **i** A(1, 5); B(0, 1)

ii $m = \frac{1-5}{0-1}$

$m = 4$

b **i** A(–1, 5); B(0, 1)

ii $m = \frac{1-5}{0-(-1)}$

$m = -4$

c **i** A(–0, 3); B(3, 2)

ii $m = \frac{2-3}{3-0}$

$m = -\frac{1}{3}$

d **i** A(0, –1); B(1, 0)

ii $m = \frac{0-(-1)}{1-0}$

$m = 1$

e **i** A(–1, –2); B(2, 0)

ii $m = \frac{0-(-2)}{2-(-1)}$

$m = \frac{2}{3}$

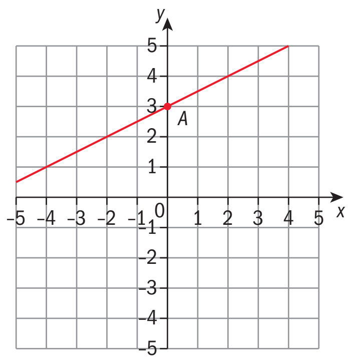
f **i** A(2, 4); B(4, 1)

$m = \frac{1-4}{4-2}$

$m = -\frac{3}{2}$

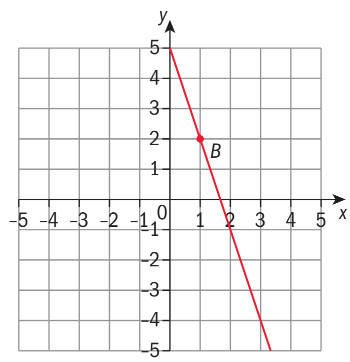
Exercise 3B

1 **a**

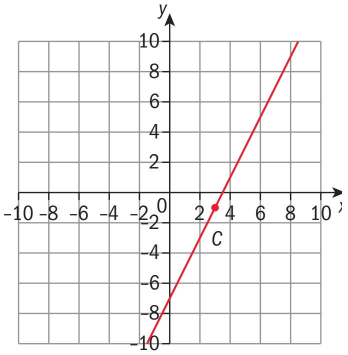


plot the given point and then using that the gradient is $m = \frac{y-\text{step}}{x-\text{step}}$ find more points lying on the line.

b



c



2 a i $m = \frac{7-5}{3-2}$

$m = 2$

ii gradient of AC = $\frac{p-5}{4-2}$

gradient of AB = 2

gradient of AC = gradient of AB

Therefore

$\frac{p-5}{4-2} = 2$

$p = 9$

b i $m = \frac{6-2}{1-0}$

$m = 4$

ii gradient of AC = $\frac{t-2}{2-0}$

gradient of AB = 4

Therefore

$\frac{t-2}{2-0} = 4$

$t = 10$

c i $m = \frac{-5-0}{1-0}$

$m = -5$

ii gradient of AC = $\frac{q-0}{2-0}$

gradient of AB = –5

Therefore

$\frac{q-0}{2-0} = -5$

$q = -10$

d i $m = \frac{0-(-1)}{1-0}$

$m = 1$

ii gradient of AC = $\frac{s-(-1)}{4-0}$

gradient of AB = 1

Therefore

$\frac{s-(-1)}{4-0} = 1$

$s = 3$

e i $m = \frac{4-1}{-6-(-5)}$

$m = -3$

ii gradient of AC = $\frac{r-1}{-4-(-5)}$

gradient of AB = –3

$\frac{r-1}{-4-(-5)} = -3$

$r = -2$

3 a $m = \frac{10-5}{a-(-1)}$

$m = \frac{5}{a+1}$

Therefore

$\frac{5}{a+1}$

b equating answer to **a** to the gradient

$4 = \frac{5}{a+1}$

$a + 1 = \frac{5}{4}$

$a = \frac{1}{4}$

4 a $m = 0.5$

b $m = \frac{t-6}{-3-2}$

$m = \frac{t-6}{-5}$

c using your answers to **a** and **b**.

$\frac{t-6}{-5} = 0.5$

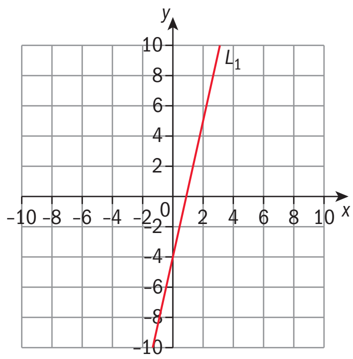
$t = 3.5$

Exercise 3C

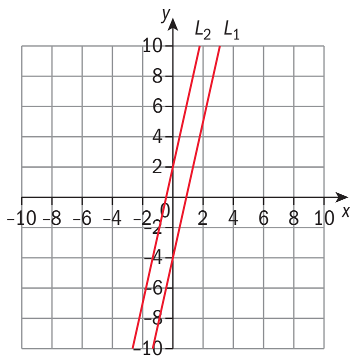
1 a $m = \frac{-4-5}{0-2}$

$m = 4.5$

b



c



2 if the y-coordinates are the same then the line is parallel to the x-axis and if the x-coordinates are the same the line is parallel to the y-axis.

a parallel to the x-axis.

b parallel to the y-axis.

c neither.

3 **a** Any horizontal line is parallel to the x...-axis.

b Any vertical line is parallel to the y...-axis.

c Any horizontal line has gradient equal to... zero...

4 If the line is parallel to the x-axis then y-coordinate of any point on that line will be always the same.

$a = 3$.

Both (5; 3) and(8, a) lie on the same line parallel to the x-axis therefore they have the same y-coordinate.

5 If the line is parallel to the y-axis then x-coordinate of any point on that line will be always the same.

$m = –5$

Both (m, 24) and (–5; 2) lie on the same line parallel to the y-axis therefore they have the same x-coordinate.

Exercise 3D

1 negative reciprocals are numbers that multiplied together give –1

a 2 and $-\frac{1}{2}$ are negative reciprocals.

b $-\frac{4}{3}$ and $\frac{3}{4}$ are negative reciprocals.

d –1 and 1 are negative reciprocals.

2 perpendicular lines have gradients that are negative reciprocals

b $\frac{4}{3}$ and $\frac{-3}{4}$ are gradients of perpendicular lines.

d 1 and –1 are gradients of perpendicular lines.

3 **a** Let $m_\perp$ be the gradient of a perpendicular line to AB.

$-3m_\perp = -1$

$m_\perp = \frac{1}{3}$

b Let $m_\perp$ be the gradient of a perpendicular line to AB.

$\frac{2}{3}m_\perp = -1$

$m_\perp = -\frac{3}{2}$ or -1.5

c Let $m_\perp$ be the gradient of a perpendicular line to AB.

$-\frac{1}{4}m_\perp = -1$

$m_\perp = 4$

d Let $m_\perp$ be the gradient of a perpendicular line to AB.

$1m_\perp = -1$

$m_\perp = -1$

e Let $m_\perp$ be the gradient of a perpendicular line to AB.

$-1m_\perp = -1$

$m_\perp = 1$

4 **a** $m = \frac{-1-6}{1-(-2)}$

$m = -\frac{7}{3}$

Let $m_\perp$ be the gradient of a perpendicular line to AB.

$m_\perp \times -\frac{7}{3} = -1$

$m_\perp = \frac{3}{7}$

b $m = \frac{-2-10}{0-5}$

$m = \frac{12}{5}$

Let $m_\perp$ be the gradient of a perpendicular line to AB.

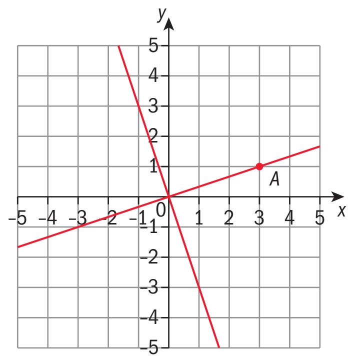
$m_\perp \times \frac{12}{5} = -1$

$m_\perp = -\frac{5}{12}$

5 **a** **i** –3

ii $\frac{1}{3}$

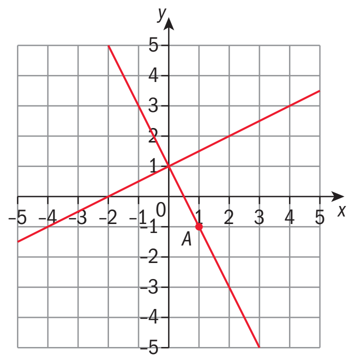
iii



b **i** $\frac{1}{2}$

ii –2

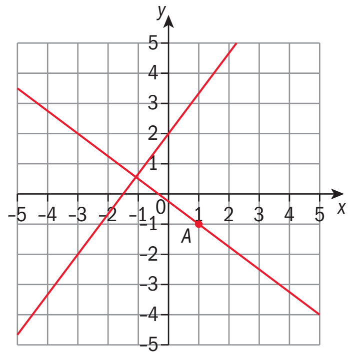
iii



c **i** $\frac{4}{3}$

ii $-\frac{3}{4}$

iii



6 **a** $m = \frac{a-3}{-2-0}$

$m = \frac{a-3}{-2}$

b $m = -\frac{1}{2}$

c Use answers to **a** and **b** to set an equation where the unknown is a.

$-\frac{1}{2} = \frac{a-3}{-2}$

$a = 4$

7 **a** $m = \frac{-8-5}{5-3}$

$m = -\frac{13}{2}$

b $m_{\perp} = \frac{2}{13}$

c $m_{\perp} = \frac{2}{13}$

$m_{\perp} = \frac{2-0}{t-5} = \frac{2}{t-5}$

$\frac{2}{13} = \frac{2}{t-5}$

$t - 5 = 13$

$t = 18$

Exercise 3E

1 **a** $y = mx + c$

$y = 3x + c$

$4 = 3 \times 1 + c$

$c = 1$

$y = 3x + 1$

b $y = mx + c$

$y = \frac{5}{3}x + c$

$8 = \frac{5}{3} \times 4 + c$

$c = \frac{4}{3}$

$y = \frac{5}{3}x + \frac{4}{3}$

c $y = mx + c$

$y = -2x + c$

$0 = -2 \times -3 + c$

$c = -6$

$y = -2x - 6$

2 **a** **i** $m = 2$

ii The point of intersection with the y-axis has the form $(0, y)$

$y = 2 \times 0 + 1$

$y = 1$

Therefore the point is $(0, 1)$

iii The point of intersection with the x-axis has the form $(x, 0)$

$y = 2x + 1$

$0 = 2x + 1$

$x = -\frac{1}{2}$

Therefore the point is $\left(-\frac{1}{2}, 0\right)$

b **i** $m = -3$

ii $y = -3 \times 0 + 2$

$y = 2$

Therefore the point is $(0, 2)$

iii $y = -3x + 2$

$0 = -3x + 2$

$x = \frac{2}{3}$

Therefore the point is $\left(\frac{2}{3}, 0\right)$

c **i** $m = -1$

ii $y = -0 + 3$

$y = 3$

Therefore the point is $(0, 3)$

iii $y = -x + 3$

$0 = -x + 3$

$x = 3$

Therefore the point is $(3, 0)$

d **i** $m = -\frac{2}{5}$

ii $y = \frac{-2}{5}x - 1$

$y = \frac{-2}{5} \times 0 - 1$

$y = -1$

Therefore the point is $(0, -1)$

iii $0 = \frac{-2}{5}x - 1$

$x = -\frac{5}{2}$

Therefore the point is $\left(-\frac{5}{2}, 0\right)$

3 **a** Expand the numerator and write the whole expression as a sum.

$y=\frac{3(x-6)}{2}$

$y=\frac{3x-18}{2}$

$y=\frac{3}{2}x-\frac{18}{2}$ or $y=1.5x-9$

b $m = 1.5$

c the y-intercept is c, $c = -9$

d The point of intersection with the x-axis has the form $(x, 0)$

$0=1.5x-9$

$x=\frac{9}{1.5}$

$x = 6$

Therefore the point is $(6, 0)$

4 **a** Using the gradient formula $m=\frac{1-(-4)}{1-2}$

$m = -5$

b $y=-5x+c$

$-4=-5\times 2 + c$

$c = 6$

$y=-5x + 6$

5 **a** $m=\frac{5-3}{2-1}$

$m = 2$

b $y=mx+c$

$y=2x+c$ substitute the gradient into the equation

$3=2\times 1 + c$ substitute P or Q in the equation to find c

$c = 1$

$y=2x+1$

c $m_{\perp}\times 2=-1$

$m_{\perp}=\frac{-1}{2}$ or -0.5

d $y=mx+c$

$y=-0.5x + c$

$2=-0.5\times 0 + c$

$c = 2$

$y=-0.5x + 2$

6 **a** $-\frac{1}{3}$

b $y = mx + c$

$1=-\frac{1}{3}\times 5 + c$

$c = \frac{8}{3}$

$y = -\frac{1}{3}x + \frac{8}{3}$

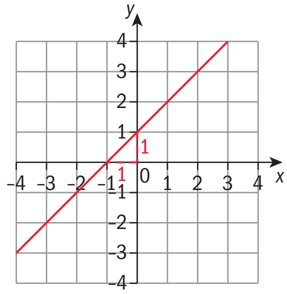
c At the x-axis the point has $y = 0$.

$y = -\frac{1}{3}x + \frac{8}{3}$

$0 = -\frac{1}{3}x + \frac{8}{3}$

$x = 8$

7 **a**



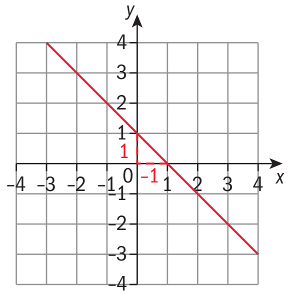
$y = mx + c$

$y = mx + 1$

$m=\frac{1}{1}$

$y = 1x + 1$

b



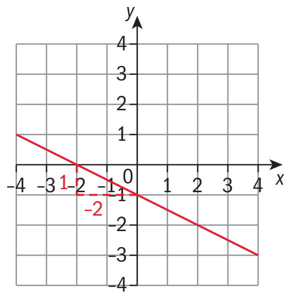
$y = mx + c$

$y = mx + 1$

$m=-\frac{1}{1}$ therefore $m=-1$

$y=-1x + 1$

c



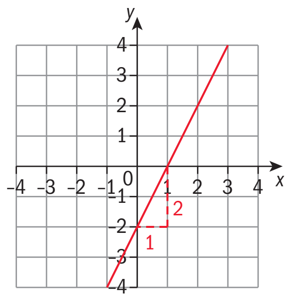
$y=mx + c$

$y=mx - 1$

$m=-\frac{1}{2}$

$y=-\frac{1}{2}x-1$

d



$y=mx + c$

$y=mx - 2$

$m=\frac{2}{1}$ therefore $m=2$

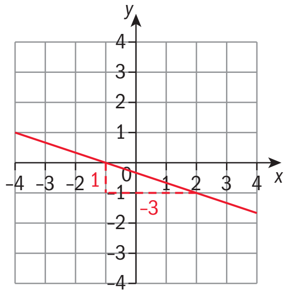
$y=2x - 2$

e two points from the graph (–1, 0) and (2, –1)

Using the formula $m = \frac{-1-0}{2-(-1)}$

$m = -\frac{1}{3}$

Or using the graph



$m = -\frac{1}{3}$

$y = -\frac{1}{3}x + c$

$0 = -\frac{1}{3} \times (-1) + c$

$c = -\frac{1}{3}$

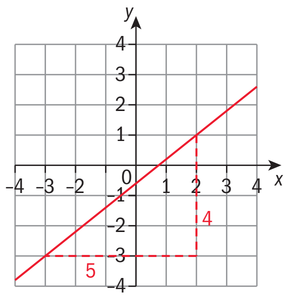
$y = -\frac{1}{3}x - \frac{1}{3}$

f two points from the graph (2, 1) and (–3, –3)

$m = \frac{-3-1}{-3-2}$

$m = \frac{4}{5}$

Or using the graph



$m = \frac{4}{5}$

$1 = \frac{4}{5} \times 2 + c$

$c = -\frac{3}{5}$

$y = \frac{4}{5}x - \frac{3}{5}$

Exercise 3F

1 a Let (x, y) be a point on this line. Substituting in the gradient formula (x, y) and $(5, 0)$.

$m = \frac{y_2 - y_1}{x_2 - x_1}$

$-4 = \frac{y-0}{x-5}$

$-4(x-5) = y$

$-4x + 20 = y$

$-4x - y + 20 = 0$ or any multiple of this equation with $a, b, d, \in \mathbb{Z}$.

b Let (x, y) be a point on this line. Substituting in the gradient formula (x, y) and $(2, 3)$

$m = \frac{y_2 - y_1}{x_2 - x_1}$

$\frac{1}{2} = \frac{y-3}{x-2}$

$1(x-2) = 2(y-3)$

$x - 2 = 2y - 6$

$x - 2y + 4 = 0$ or any multiple of this equation with $a, b, d, \in \mathbb{Z}$.

c $m = \frac{3-(-2)}{-1-3}$

$m = -\frac{5}{4}$

Let (x, y) be a point on this line.

Substituting in the gradient formula (x, y) and $(3, -2)$

$-\frac{5}{4} = \frac{y-(-2)}{x-3}$

$-5(x-3) = 4(y+2)$

$-5x + 15 = 4y + 8$

$5x + 4y - 7 = 0$ or any multiple of this equation with $a, b, d, \in \mathbb{Z}$.

d $A(0, 5)$ and $B(-5, 0)$.

$m = \frac{0-5}{-5-0}$

$m = 1$

$1 = \frac{y-0}{x-(-5)}$

$x + 5 = y$

$x - y + 5 = 0$ or any multiple of this equation with $a, b, d, \in \mathbb{Z}$.

2 Make y the subject of the formula.

a Make y the subject of the formula. $3x + y = 0$

$y = -3x$

b $x + y + 1 = 0$

$y = -x - 1$

c $2x + y - 1 = 0$

$y = -2x + 1$

d $2x - 4y = 0$

$y = \frac{-2x}{-4}$

$y = \frac{1}{2}x$ or $y = 0.5x$

e $6x + 3y - 9 = 0$

$y = \frac{-6x+9}{3}$

$y = \frac{-6x}{3} + \frac{9}{3}$

$y = -2x + 3$

3 **a** Make y the subject of the formula.

$3x-6y+6=0$

$y=\frac{3x+6}{6}$

$y=\frac{3x}{6}+\frac{6}{6}$

$y=\frac{1}{2}x+1$ or $y=0.5x+1$

b At the x-intercept the y-coordinate is 0.

$y=\frac{1}{2}x+1$

$0=\frac{1}{2}x+1$

$x=-2$

c The y-intercept is c. $y = 1$

4 **a** Point A(3, 0)

$y=2x-6$

$y=2\times3-6$

$y=0$

When $x = 3$, $y=0$ therefore the point A lies on this line.

Point B(0, 3)

$y=2x-6$

$y=2\times0-6$

$y=-6$

When $x = 0$ the value of y is not 3 therefore the point B does not lie on this line.

Point C(1, –4)

$y=2x-6$

$y=2\times1-6$

$y=-4$

When $x=1$, $y=-4$ therefore the point C lies on this line.

Point D(4, 2)

$y=2x-6$

$y=2\times4-6$

$y=2$

When $x=4$, $y=2$ therefore the point D lies on this line.

Point E(10,12)

$y=2x-6$

$y=2\times10-6$

$y=14$

When $x=10$, the value of y is not 12 therefore the point E does not lie on this line.

Point F(5, 4)

$y=2x-6$

$y=2\times5-6$

$y=4$

When $x=5$, $y=4$ therefore the point F lies on this line.

b $y=2x-6$

$7=2a-6$

$a=\frac{13}{2}$ or $a = 6.5$

c $y=2x-6$

$t=2\times7-6$

$t=8$

5 **a** Point A(1, 4)

$-6x+2y-2=0$

$-6\times1+2\times4-2=0$

$0=0$

Therefore point A lies on this line.

Point B(0, 1)

$-6x+2y-2=0$

$-6\times0+2\times1-2=0$

$0=0$

Therefore point B lies on this line.

Point C(1, 0)

$-6x+2y-2=0$

$-6\times1+2\times0-2=0$

$-8=0$ which is not true therefore point C does not lie on this line.

Point D(2, 6)

$-6x+2y-2=0$

$-6\times2+2\times6-2=0$

$-2=0$ which is not true therefore point D does not lie on this line.

Point E$\left(-\frac{1}{3}, 0\right)$

$-6x+2y-2=0$

$-6\times\left(-\frac{1}{3}\right)+2\times0-2=0$

$0=0$

Therefore point E lies on this line.

b $-6x+2y-2=0$

$-6a+2\times3-2=0$

$a=\frac{2}{3}$

c $-6x+2y-2=0$

$-6\times10+2t-2=0$

$t=31$

6 There several ways to solve this question. One of them is to choose one line and see which of the conditions described in the second column verifies.

A: $6x-3y+15=0$

We write the equation in the form $y=mx+c$

$6x-3y+15=0$

$3y = 6x+15$

$y=\frac{6x+15}{3}$

$y=2x+5$

The gradient is 2 and the y-intercept is 5 therefore it matches with **H**.

B: $y=2x-5$

The gradient is 2 therefore it is not F and the y-intercept is –5 therefore it is not E. It is **G**.

C: $10x+5y+25=0$

"The x-intercept is 2.5" means that the line passes through the point (2.5, 0). Substitute (2.5, 0) in the given equation.

$10 \times 2.5 + 5 \times 0 + 25 = 0$

$50 \neq 0$ therefore (2.5,0) does not lie on this line. Therefore it is not E and so it is F.

D: $y=-2x+5$

It is E. The y-intercept is 5 and when x is 2.5 the value of y is 0.

> make y the subject of the formula.

7 **a** $2x-y+6=0$

$y=2x+6$

The gradient of L_1 is 2.

b The y-intercept of L_1 is 6.

c substitute into the equation

$1.5 = 2c+6$

$c = -2.25$

d $t=2 \times 5+6$

$t=11$

e parallel lines have equal gradients so the gradient of L_1 is 2.

> if it passes through C(0, 4) then the y-intercept is 4.

f $y = 2x + 4$

8 **a** $m = \frac{6-2}{-1-1}$

$m = -2$

$y = -2x + c$

Using the point A(1, 2)

$2 = -2 \times 1 + c$

$c = 4$

$y = -2x + 4$

b Points are collinear if they lie on the same line. Putting the coordinates of C in the equation of the line gives $y = -2 \times 10 + 4$

$y = -16$

Therefore C lies on this line and A, B and C are collinear.

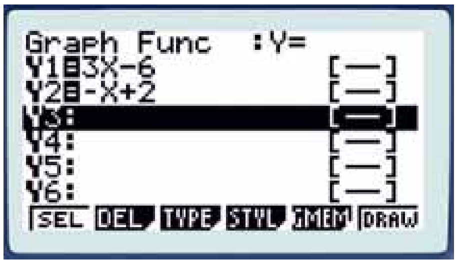
Exercise 3G

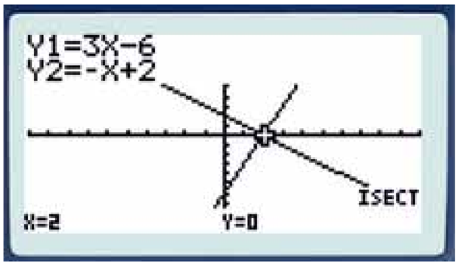
1 **i** Vertical lines have equations of the form $x=k$ $\quad x=3$

ii Horizontal lines have equations of the form $y=k$ $\quad y=1$

2 **a**

> use your GDC. In the graph mode input both equations and find the intersection point.





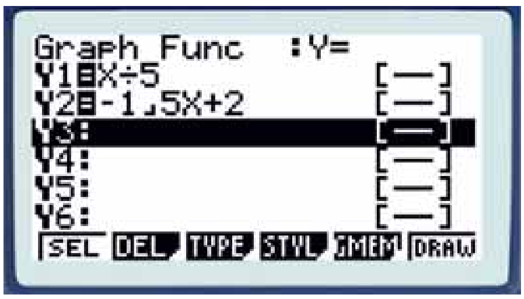
The intersection point is (2, 0).

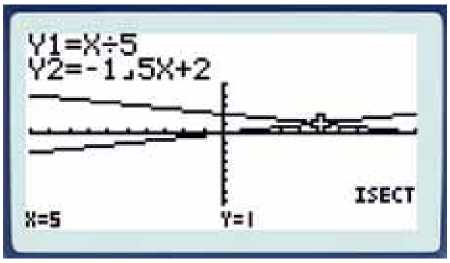
b **Method 1:** write down both equations in the form $y=mx+c$ and then use the GDC as shown in **a**.

$-x+5y=0 \Rightarrow y=\frac{x}{5}$

and

$\frac{1}{5}x+y-2=0 \Rightarrow y=-\frac{1}{5}x+2$





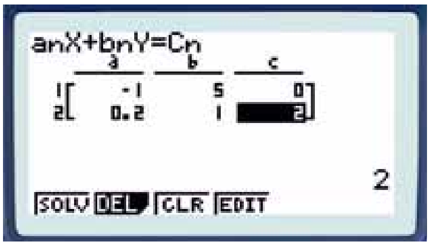
The intersection point is (5, 1).

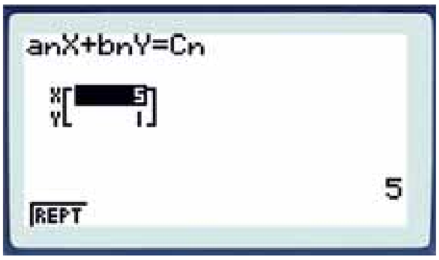
Method 2: solve the simultaneous equations in the equations mode.

$-x+5y=0$

$\frac{1}{5}x+y-2=0 \Rightarrow \frac{1}{5}x+y=2$

The intersection point is (5, 1)

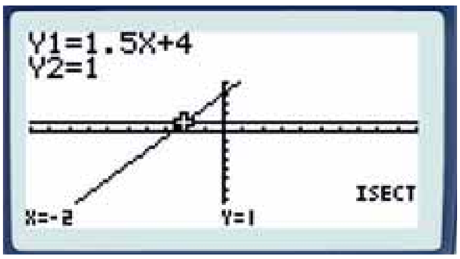




c The intersection point is (–7, 3).

d GDC.

$y = 1.5x + 4$ and $y = 1$



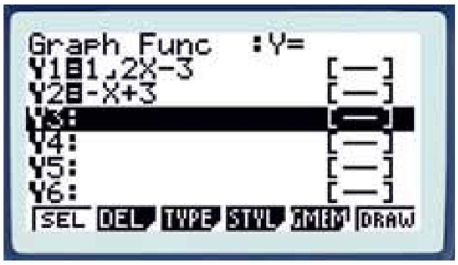
The intersection point is (–2, 1)

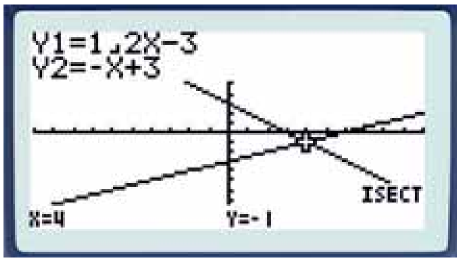
e **Method 1:** write down both equations in the form $y=mx+c$ and then use the GDC

$-x+2y+6=0 \Rightarrow y=\frac{x-6}{2} \Rightarrow y=\frac{1}{2}x-3$

and

$x+y-3=0 \Rightarrow y=-x+3$



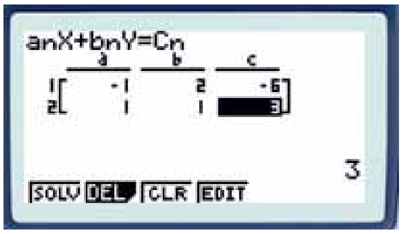


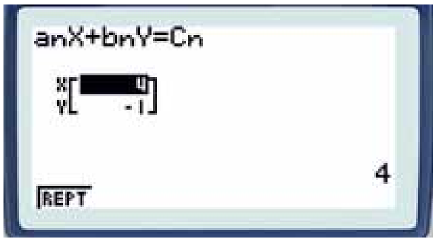
The intersection point is (4,–1)

Method 2: solve the simultaneous equations in the GDC equations mode.

$-x+2y+6=0 \Rightarrow -x+2y=-6$

$x+y-3=0 \Rightarrow x+y=3$





The intersection point is (4, –1)

f The lines are $x=0$ and $y=4$

The intersection point is (0, 4)

3 Write the equations in the form $y=mx+c$ and compare the gradients.

$L_1: -5x+y+1=0 \Rightarrow y=5x-1$

and

$L_2: 10x-2y+4=0 \Rightarrow y=\frac{10x+4}{2} \Rightarrow y=5x+2$

Gradient of L_1 = gradient of L_2 = 5

Therefore both lines are parallel.

4 **a** $y=3(x-5) \Rightarrow y=3x-15$

and

$x-\frac{1}{3}y+6=0 \Rightarrow \frac{1}{3}y=x+6 \Rightarrow y=3x+18$

Both gradients are equal and they have different y-intercept therefore these lines do not meet at any point.

b $\frac{y+1}{x-2} = -1 \Rightarrow y+1 = -1(x-2) \Rightarrow y = -x+1$
and
$y = -x + 1$
They are the same line (same gradient and same y-intercept) therefore they meet at an infinite number of points.

c $y = 4x - 8$
and
$4x - 2y = 0 \Rightarrow y = 2x$
They have different gradients (4 and 2) and different y-intercepts (–8 and 0) therefore they meet at only one point.

d $x - y + 3 = 0 \Rightarrow y = x + 3$
and
$3x - 3y + 9 = 0 \Rightarrow y = \frac{3x+9}{3} \Rightarrow y = x + 3$
They are the same line (same gradient and same y-intercept) therefore they meet at an infinite number of points.

5 **a** Point A lies on both lines.
$y = 5x + c$
A(1, 0) lies on L_1
$0 = 5 \times 1 + c$
$c = -5$
$y = 5x - 5$

b Gradient of $L_2 = -\frac{1}{5}$
$y = -\frac{1}{5}x + c$
A(1, 0) lies on L_2
$0 = -\frac{1}{5} \times 1 + c$
$c = \frac{1}{5}$
$y = -\frac{1}{5}x + \frac{1}{5}$

2 **a** $\cos\delta = \frac{AC}{AB}$; $\sin\delta = \frac{BC}{AB}$; $\tan\delta = \frac{BC}{AC}$

b $\cos\delta = \frac{QR}{PQ}$; $\sin\delta = \frac{PR}{PQ}$; $\tan\delta = \frac{PR}{QR}$

c $\cos\delta = \frac{EF}{DF}$; $\sin\delta = \frac{ED}{DF}$; $\tan\delta = \frac{ED}{EF}$

3 find first the missing side.

a $hyp^2 = 5^2 + 4^2$
$hyp = \sqrt{41}$

i $\sin\alpha = \frac{opp}{hyp}$
$\sin\alpha = \frac{4}{\sqrt{41}}$

ii $\cos\alpha = \frac{adj}{hyp}$
$\cos\alpha = \frac{5}{\sqrt{41}}$

iii $\tan\alpha = \frac{opp}{adj}$
$\tan\alpha = \frac{4}{5}$

b $6^2 + opp^2 = 8^2$
$opp^2 = 8^2 - 6^2$
$opp = \sqrt{28}$

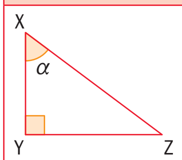
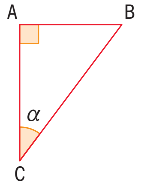
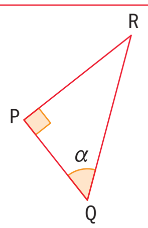
i $\sin\alpha = \frac{opp}{hyp}$
$\sin\alpha = \frac{\sqrt{28}}{8}$

ii $\cos\alpha = \frac{adj}{hyp}$
$\cos\alpha = \frac{6}{8}$

iii $\tan\alpha = \frac{opp}{adj}$
$\tan\alpha = \frac{\sqrt{28}}{6}$

Exercise 3H

1

Triangle	Hypotenuse	Side opposite ∝	Side adjacent to ∝
	XZ	YZ	XY
	CB	AB	AC
	RQ	PR	PQ

c $10^2 + adj^2 = 14^2$

$adj^2 = 14^2 - 10^2$

$adj = \sqrt{96}$

i $\sin\alpha = \frac{opp}{hyp}$

$\sin\alpha = \frac{10}{14}$

ii $\cos\alpha = \frac{adj}{hyp}$

$\cos\alpha = \frac{\sqrt{96}}{14}$

iii $\tan\alpha = \frac{opp}{adj}$

$\tan\alpha = \frac{10}{\sqrt{96}}$

4 **a** $\sin\beta = \frac{x}{10}$ **b** $\cos\beta = \frac{x}{5}$

c $\tan\beta = \frac{x}{12}$ **d** $\tan\beta = \frac{7}{x}$

e $\sin\beta = \frac{14}{x}$ **f** $\cos\beta = \frac{3}{x}$

Exercise 3I

1 $\tan 46° = \frac{h}{3}$

$h = 3.11$ (3 s.f.)

2 $\cos 20.5° = \frac{6}{x}$

$x = \frac{6}{\cos 20.5°}$

$x = 6.41$ (2 d.p)

3 $\tan 26° = \frac{m}{10}$

$m = 4.88$ (2 d.p)

4 $\sin 40.2° = \frac{9}{y}$

$y = \frac{9}{\sin 40.2°}$

$y = 13.94$ (2 d.p)

5 $\sin 15° = \frac{100}{t}$

$t = \frac{100}{\sin 15°}$

$t = 386.37$ (2 d.p)

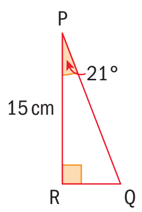
6 $\tan 30° = \frac{50}{s}$

$s = \frac{50}{\tan 30°}$

$s = 86.60$ (2 d.p)

Exercise 3J

1 **a**



b Sum of the interior angles of a triangle *is* 180°.

$\hat{Q} + 90° + 21° = 180°$

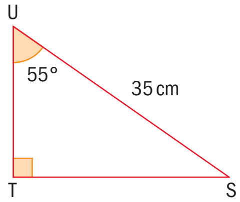
$\hat{Q} = 69°$

c $\tan 21° = \frac{QR}{PR}$

$\tan 21° = \frac{QR}{15}$

$QR = 5.76$ cm (3 s.f.)

2 **a**



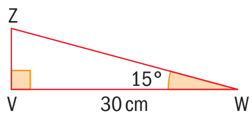
b $\hat{S} + 90° + 55° = 180°$

$\hat{S} = 35°$

c $\cos 55° = \frac{TU}{35}$

$TU = 20.1$ cm (3 s.f.)

3 **a**



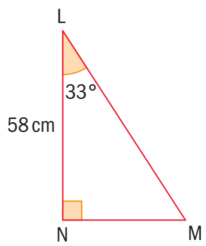
b $\hat{Z} + 90° + 15° = 180°$

$\hat{Z} = 75°$

c $\tan 15° = \frac{VZ}{30}$

$VZ = 8.04$ cm (3 s.f)

4 **a**



b $\hat{M} + 90° + 33° = 180°$

$\hat{M} = 57°$

c $\cos 33° = \frac{58}{LM}$

$LM = \frac{58}{\cos 33°}$

$LM = 69.2$ cm (3 s.f)

5 **a** BCD is right-angled triangle.

$\tan 30° = \frac{BC}{12}$

$BC = 6.93$ cm (3 s.f.)

b Perimeter of the rectangle $ABCD = 2DC + 2BC$

Perimeter of the rectangle $ABCD = 2 \times 12 + 2 \times 6.9282.....$

Perimeter of the rectangle $ABCD = 37.9$ cm (3 s.f.)

c Area of the rectangle $ABCD = DC \times BC$

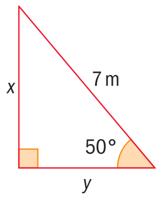
Area of the rectangle $ABCD = 12 \times 6.9282....$

Area of the rectangle $ABCD = 83.1$ cm^2 (3 s.f.)

6 $\tan 46° = \frac{h}{7}$

$h = 7.25$ m (3 s.f.)

7 **a**



b $\sin 50° = \frac{x}{7}$

$x = 5.63$ m (3 s.f.)

c $\cos 50° = \frac{y}{7}$

$y = 4.50$ m (3 s.f.)

Exercise 3K

1 **a** $\sin^{-1}(0.6)$ means the angle with a sine of 0.6

b $\tan^{-1}\left(\frac{1}{2}\right)$ means the angle with a tangent of $\frac{1}{2}$

c $\cos^{-1}\left(\frac{2}{3}\right)$ means the angle with a cosine of $\frac{2}{3}$

2 use your GDC.

a $\sin^{-1}(0.6) = 36.9°$

b $\tan^{-1}\left(\frac{1}{2}\right) = 26.6°$

c $\cos^{-1}\left(\frac{2}{3}\right) = 48.2°$

3 **a** $\sin\alpha = 0.2$

$\alpha = \sin^{-1} 0.2$

$\alpha = 11.5°$

b $\cos\alpha = \frac{2}{3}$

$\alpha = \cos^{-1}\left(\frac{2}{3}\right)$

$\alpha = 48.2°$

c $\tan\alpha = 1$

$\alpha = \tan^{-1} 1$

$\alpha = 45°$

4 **a** $\tan A = \frac{9.5}{7}$

$A = \tan^{-1}\left(\frac{9.5}{7}\right)$

$A = 53.6°$

$C = 180 - 90 - 53.61....$

$C = 36.4°$

b $\cos R = \frac{6}{8}$

$R = \cos^{-1}\left(\frac{6}{8}\right)$

$R = 41.4°$

$C = 180 - 90 - 41.4096...$

$C = 48.6°$

c $\cos M = \frac{10}{12.5}$

$M = \cos^{-1}\left(\frac{10}{12.5}\right)$

$M = 36.9°$

$C = 180 - 90 - 36.869...$

$C = 53.1°$

d $\sin Z = \frac{150}{200}$

$Z = \sin^{-1}\left(\frac{150}{200}\right)$

$Z = 48.6°$

$Y = 180 - 90 - 48.5903...$

$Y = 41.4°$

e $\tan J = \frac{7.2}{2.6}$

$J = \tan^{-1}\left(\frac{7.2}{2.6}\right)$

$J = 70.1°$

$I = 180 - 90 - 70.144...$

$I = 19.9°$

f $\cos F = \frac{3.5}{8}$

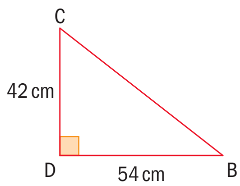
$F = \cos^{-1}\left(\frac{3.5}{8}\right)$

$F = 64.1°$

$E = 180 - 90 - 64.0555...$

$E = 25.9°$

5 **a**

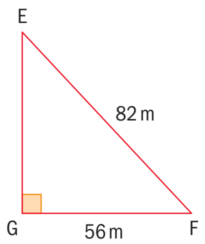


b $\tan C = \frac{54}{42}$

$C = \tan^{-1}\left(\frac{54}{42}\right)$

$C = 52.1°$

6 a

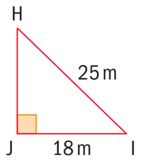


b $\cos F = \frac{56}{82}$

$F = \cos^{-1}\left(\frac{56}{82}\right)$

$F = 46.9°$

7 a



b $\sin H = \frac{18}{25}$

$H = \sin^{-1}\left(\frac{18}{25}\right)$

$H = 46.1°$

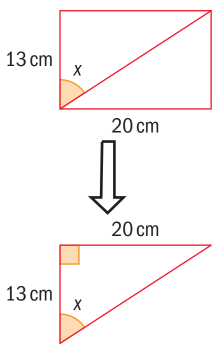
8 BCD is a right-angled triangle.

$\tan BDC = \frac{5}{10}$

$BDC = \tan^{-1}\left(\frac{5}{10}\right)$

$BDC = 26.6°$ (3 s.f.)

9

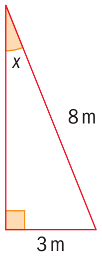


$\tan x = \frac{20}{13}$

$x = \tan^{-1}\left(\frac{20}{13}\right)$

$x = 57.0°$ (3 s.f.)

10

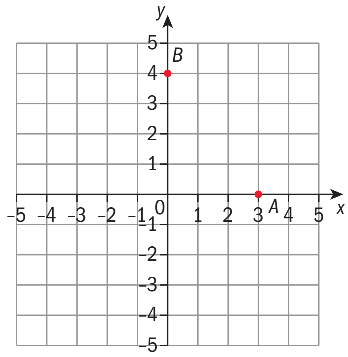


$\sin x = \frac{3}{8}$

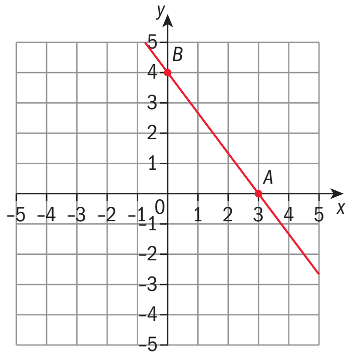
$x = \sin^{-1}\left(\frac{3}{8}\right)$

$x = 22.0°$ (3 s.f.)

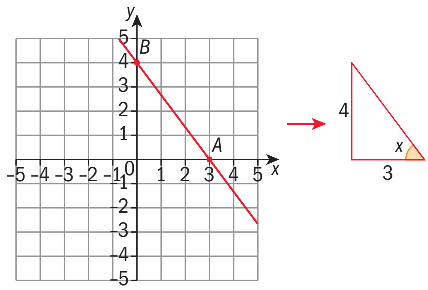
11 a



b



c

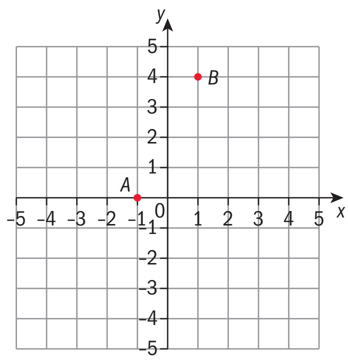


$\tan x = \frac{4}{3}$

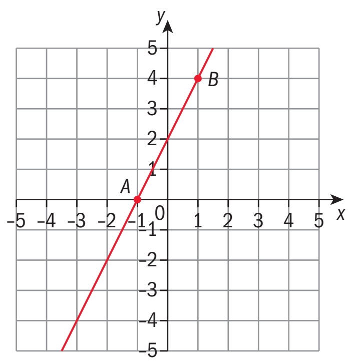
$x = \tan^{-1}\left(\frac{4}{3}\right)$

$x = 53.1°$

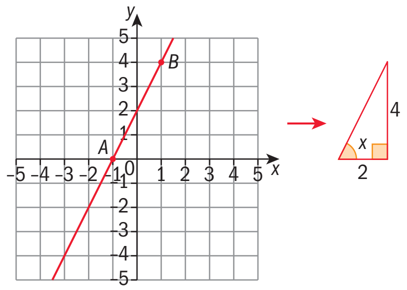
12 a



b



c



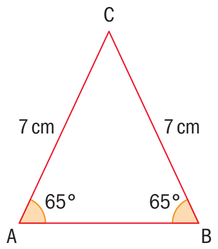
$\tan x = \frac{4}{2}$

$x = \tan^{-1}\left(\frac{4}{2}\right)$

$x = 63.4°$

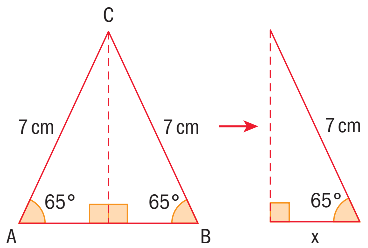
Exercise 3L

1 a



The height of triangle ABC bisects AB (and is perpendicular to AB).

b



Let x be half of AB.

$\cos 65° = \frac{x}{7}$

$x = 2.958...$ cm

$AB = 2 \times x$

$AB = 2 \times 2.958...$

$AB = 5.92$ cm

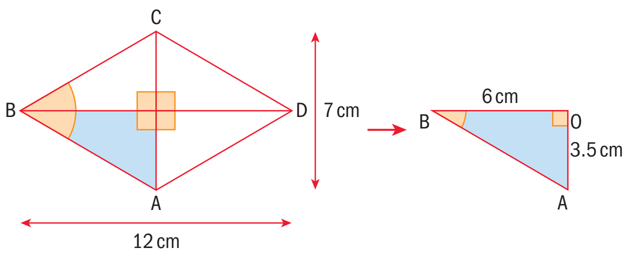
c Perimeter of $ABC = AB + BC + CA$

Perimeter of $ABC = AB + 2 \times BC$

Perimeter of $ABC = 5.92 + 2 \times 7$

Perimeter of $ABC = 19.9$ cm $= 20$ cm correct to the nearest cm.

2



ABC is the requested angle. We first find ABO which is half of ABC.

$\tan ABO = \frac{3.5}{6}$ $ABO = \tan^{-1}\left(\frac{3.5}{6}\right)$

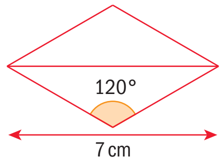
$ABO = 30.25...$

$ABC = 2 \times ABO$

$ABC = 2 \times 30.25...$

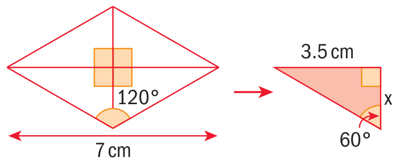
$ABC = 60.5°$

3 a



draw the shorter diagonal and remember that the two diagonals of the rhombus bisect each other at right-angles.

b



Let x be half of the shorter diagonal.

$\tan 60° = \frac{3.5}{x}$

$x = \frac{3.5}{\tan 60°}$

$x = 2.0207...$

$2x = 4.04$ cm (3 s.f.)

drop a perpendicular to DC from B.

4 a $DE = \frac{16-12}{2}$

$DE = 2$ m

b $\cos D = \frac{2}{6}$

$D = \cos^{-1}\left(\frac{2}{6}\right)$

$D = 70.5°$ (3 s.f.)

5 a Drop a perpendicular from Q to SR. Let T be the point of where the perpendicular and SR meet. Apply Pythagoras in QTR.

$3^2 + QT^2 = 5^2$

$QT^2 = 5^2 - 3^2$

$QT^2 = 16$

$SP = QT = 4$ cm

b Area of PQRS $= \frac{4}{2}(10+7)$

Area of PQRS $= 34$ cm^2

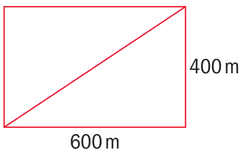
c $\cos SRQ = \frac{3}{5}$

$SRQ = \cos\left(\frac{3}{5}\right)^{-1}$

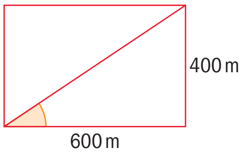
$SRQ = 53.1°$ (3 s.f.)

sine or tangent can also be used.

6 **a**



b

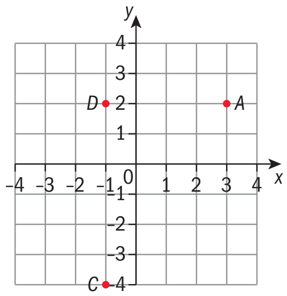


$\sin x = \frac{400}{600}$

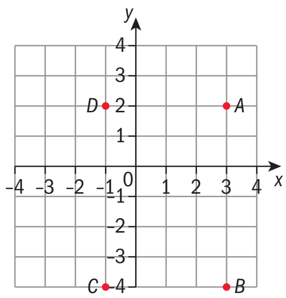
$x = \sin^{-1}\left(\frac{400}{600}\right)$

$x = 41.8°$

7 **a**



b **i**

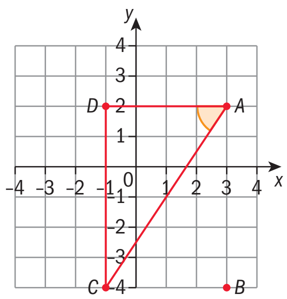


ii (3,–4)

c **i** AB = 6

ii BC = 4

d



Let x be the required angle.

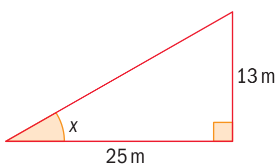
$\tan x = \frac{6}{4}$

$x = \tan^{-1}\left(\frac{6}{4}\right)$

$x = 56.3°$ (3 s.f.)

Exercise 3M

1

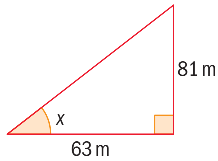


$\tan x = \frac{13}{25}$

$x = \tan^{-1}\left(\frac{13}{25}\right)$

$x = 27.5°$

2

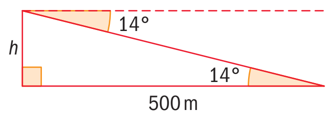


$\tan x = \frac{81}{63}$

$x = \tan^{-1}\left(\frac{81}{63}\right)$

$x = 52.1°$ (3 s.f.)

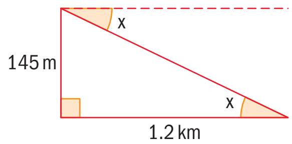
3



$\tan 14° = \frac{h}{500}$

$h = 125$ m (3 s.f.)

4 Put all the measurements in the same unit.



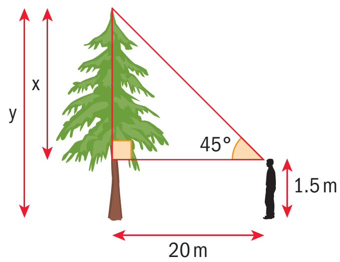
1.2 km = 1200 m

$\tan x = \frac{145}{1200}$

$x = \tan^{-1}\left(\frac{145}{1200}\right)$

$x = 6.89°$ (3 s.f.)

5



Let y be the height of the tree. The $y = x + 1.5$

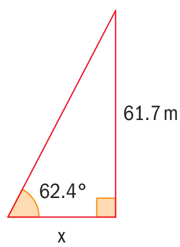
$\tan 45° = \frac{x}{20}$

$x = 20$ m

$y = 20 + 1.5$

$y = 21.5$ m

6

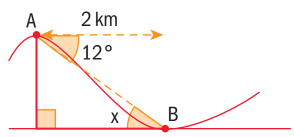


$\tan 62.4° = \frac{61.7}{x}$

$x = \frac{61.7}{\tan 62.4°}$

$x = 32.3$ m

7 **a**



$x = 12°$

As both x and 12° angles are alternate interior angles.

b The vertical distance is the length of the side opposite angle x (found in **a**).

Let y be the required distance.

$\tan 12° = \frac{y}{2}$

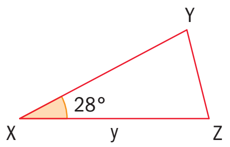
$y = 0.4251\ldots$ km

$y = 425$ m (nearest metre)

Exercise 3N

1 Substitute into the sine rule formula.

a

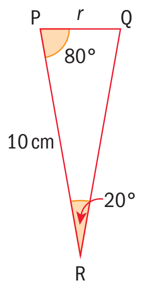


$$\frac{y}{\sin 67^\circ} = \frac{7}{\sin 28^\circ}$$

$$y = \frac{7\sin 67^\circ}{\sin 28^\circ}$$

$$y = 13.7 \text{ km (3 s.f.)}$$

b



$$\frac{r}{\sin 20^\circ} = \frac{10}{\sin Q}$$

$$Q = 180° - 20° - 80°$$

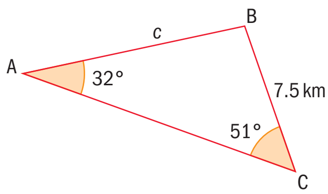
$$Q = 80°$$

$$\frac{r}{\sin 20^\circ} = \frac{10}{\sin 80^\circ}$$

$$r = \frac{10\sin 20^\circ}{\sin 80^\circ}$$

$$r = 3.47 \text{ cm (3 s.f.)}$$

c

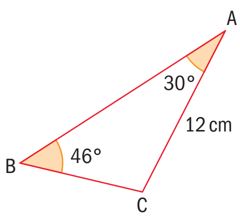


$$\frac{c}{\sin 51^\circ} = \frac{7.5}{\sin 32^\circ}$$

$$c = \frac{7.5\sin 51^\circ}{\sin 32^\circ}$$

$$c = 11.0 \text{ km (3 s.f.)}$$

2

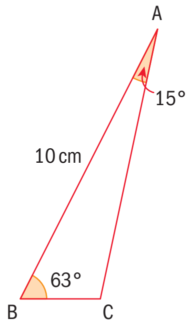


$$\frac{BC}{\sin 30^\circ} = \frac{12}{\sin 46^\circ}$$

$$BC = \frac{12\sin 30^\circ}{\sin 46^\circ}$$

$$BC = 8.34 \text{ cm}$$

3



$$\frac{BC}{\sin 15^\circ} = \frac{10}{\sin C}$$

$$C = 180° - 63° - 15°$$

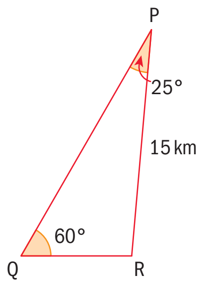
$$C = 102°$$

$$\frac{BC}{\sin 15^\circ} = \frac{10}{\sin 102^\circ}$$

$$BC = \frac{10\sin 15^\circ}{\sin 102^\circ}$$

$$BC = 2.65 \text{ cm (3 s.f.)}$$

4



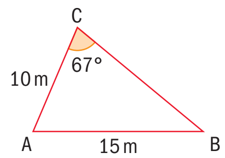
$$\frac{QR}{\sin 25^\circ} = \frac{15}{\sin 60^\circ}$$

$$QR = \frac{15\sin 25^\circ}{\sin 60^\circ}$$

$$QR = 7.32 \text{ km (3 s.f.)}$$

5 Substitute into the sine rule formula.

a



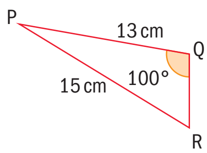
$$\frac{15}{\sin 67^\circ} = \frac{10}{\sin B}$$

$$\sin B = \frac{10\sin 67^\circ}{15}$$

$$B = \sin^{-1}\left(\frac{10\sin 67^\circ}{15}\right)$$

$$B = 37.9° \text{ (3 s.f.)}$$

b



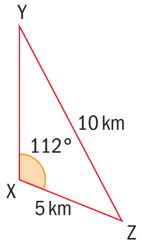
$$\frac{15}{\sin 100^\circ} = \frac{13}{\sin R}$$

$$\sin R = \frac{13\sin 100^\circ}{15}$$

$$R = \sin^{-1}\left(\frac{13\sin 100^\circ}{15}\right)$$

$$R = 58.6° \text{ (3 s.f.)}$$

c



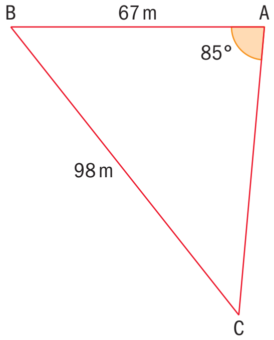
$$\frac{10}{\sin 112^\circ} = \frac{5}{\sin Y}$$

$$\sin Y = \frac{5\sin 112^\circ}{10}$$

$$Y = \sin^{-1}\left(\frac{5\sin 112^\circ}{10}\right)$$

$Y = 27.6°$ (3 s.f.)

6



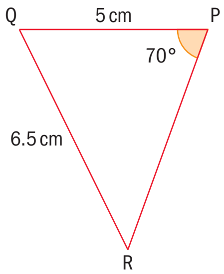
$$\frac{98}{\sin 85^\circ} = \frac{67}{\sin C}$$

$$\sin C = \frac{67\sin 85^\circ}{98}$$

$$C = \sin^{-1}\left(\frac{67\sin 85^\circ}{98}\right)$$

$C = 42.9°$ (3 s.f.)

7



$$\frac{6.5}{\sin 70^\circ} = \frac{5}{\sin R}$$

$$\sin R = \frac{5\sin 70^\circ}{6.5}$$

$$R = \sin^{-1}\left(\frac{5\sin 70^\circ}{6.5}\right)$$

$R = 46.3°$ (3 s.f.)

8 **a** $BCX = 180° - 30°$

$BCX = 150°$

b using triangle CBX $\frac{CB}{\sin 10^\circ} = \frac{10}{\sin CBX}$

$CBX = 180° - 150° - 10°$

$CBX = 20°$

$$\frac{CB}{\sin 10^\circ} = \frac{10}{\sin 20^\circ}$$

$$CB = \frac{10\sin 10^\circ}{\sin 20^\circ}$$

$CB = 5.08$ m (3 s.f.)

c using triangle ABC $\sin 30° = \frac{AB}{BC}$

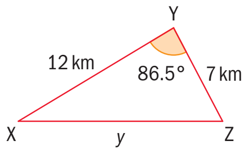
$$\sin 30° = \frac{AB}{5.0771\ldots}$$

$AB = 2.54$ m (3 s.f.)

Exercise 30

1 Using cosine rule formula

a

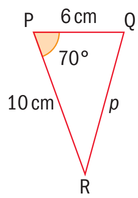


$y^2 = 12^2 + 7^2 - 2 \times 12 \times 7 \times \cos 86.5°$

$y^2 = 182.74\ldots$

$y = 13.5$ km (3 s.f.)

b

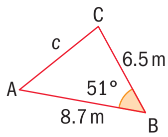


$p^2 = 6^2 + 10^2 - 2 \times 6 \times 10 \times \cos 70°$

$p^2 = 94.957\ldots$

$p = 9.74$ cm (3 s.f.)

c



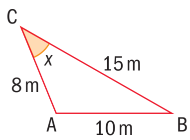
$c^2 = 8.7^2 + 6.5^2 - 2 \times 8.7 \times 6.5 \times \cos 51°$

$c^2 = 46.7638\ldots$

$c = 6.84$ m (3 s.f.)

2 In these cases we are looking for angles.

a



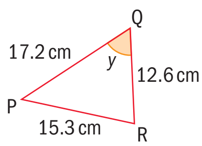
$$\cos x = \frac{8^2 + 15^2 - 10^2}{2 \times 8 \times 15}$$

$\cos x = 0.7875$

$x = \cos^{-1}(0.7875)$

$x = 38.0°$ (3 s.f.)

b



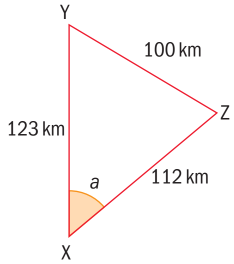
$$\cos y = \frac{17.2^2 + 12.6^2 - 15.3^2}{2 \times 17.2 \times 12.6}$$

$\cos y = 0.50874\ldots$

$y = \cos^{-1}(0.50874\ldots)$

$y = 59.4°$ (3 s.f.)

c



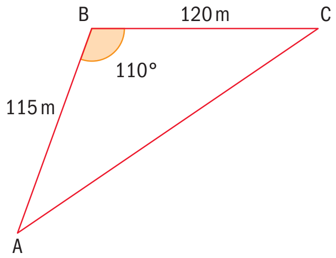
$$\cos a = \frac{112^2 + 123^2 - 100^2}{2 \times 112 \times 123}$$

$$\cos a = 0.6414...$$

$$a = \cos^{-1}(0.6414...)$$

$$a = 50.1° \text{ (3 s.f.)}$$

3

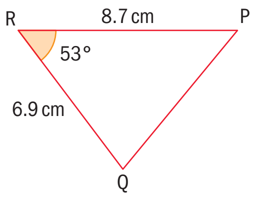


$$AC^2 = 120^2 + 115^2 - 2 \times 120 \times 115 \times \cos 110°$$

$$AC^2 = 37064.755...$$

$$AC = 193 \text{ m (3 s.f.)}$$

4

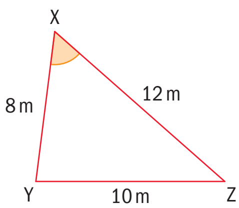


$$PQ^2 = 6.9^2 + 8.7^2 - 2 \times 6.9 \times 8.7 \times \cos 53°$$

$$PQ^2 = 51.046...$$

$$PQ = 7.14 \text{ cm (3 s.f.)}$$

5



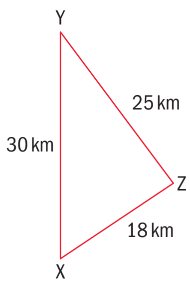
$$\cos X = \frac{8^2 + 12^2 - 10^2}{2 \times 8 \times 12}$$

$$\cos X = 0.5625$$

$$X = \cos^{-1}(0.5625)$$

$$X = 55.8° \text{ (3 s.f.)}$$

6 a



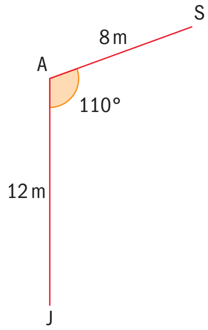
b $$\cos Z = \frac{25^2 + 18^2 - 30^2}{2 \times 25 \times 18}$$

$$\cos Z = 0.05444...$$

$$Z = \cos^{-1}(0.05444...)$$

$$X = 86.9° \text{ (3 s.f.)}$$

7 a

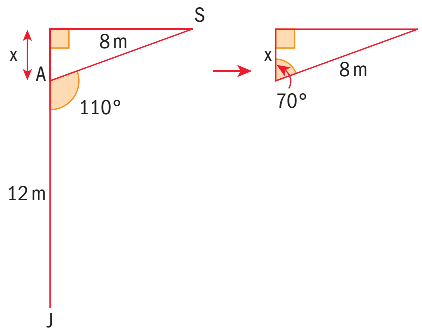


b $$SJ^2 = 12^2 + 8^2 - 2 \times 8 \times 12 \times \cos 110°$$

$$SJ^2 = 273.66....$$

$$SJ = 16.5 \text{ m (3 s.f.)}$$

c Extend the line AJ and draw a perpendicular from S to AJ.



$$\cos 70° = \frac{x}{8}$$

$$x = 2.74 \text{ m (3 s.f.)}$$

AO = BO

8 $$\cos AOB = \frac{3^2 + 3^2 - 5^2}{2 \times 3 \times 3}$$

$$\cos AOB = -0.38888....$$

$$AOB = \cos^{-1}(-0.38888....)$$

$$AOB = 113° \text{ (3 s.f.)}$$

9 a In triangle PQR,

$$PR^2 = 8.2^2 + 12.3^2 - 2 \times 8.2 \times 12.3 \times \cos 100°$$

$$PR^2 = 253.558...$$

$$PR = 15.9 \text{ m (3 s.f.)}$$

b you can apply either sine rule or cosine rule.

$$\frac{15.9235...}{\sin 100°} = \frac{8.2}{\sin PRQ}$$

$$\sin PRQ = \frac{8.2 \sin 100°}{15.9235...}$$

$$\sin PRQ = 0.50713...$$

$$PRQ = \sin^{-1}(0.50713...)$$

$$PRQ = 30.5° \text{ (3 s.f.)}$$

c QPR= $180^0 - 100^0 - 30.5^0$

QPR=49.5^0

RPM= $90^0 - 49.5^0$

RPM= 40.5^0

$\sin RPM = \frac{7.8+h}{PR}$

$\sin 40.5° = \frac{7.8+h}{15.9235...}$

$10.34... = 7.8 + h$

$h = 2.54$ m (3 s.f.)

Exercise 3P

1 Use the area of a triangle formula.

a $A = \frac{1}{2} \times 12 \times 7 \times \sin 82°$

$A = 41.6$ km^2 (3 s.f.)

b $A = \frac{1}{2} \times 81.7 \times 60.5 \times \sin 50°$

$A = 1890$ m^2 (3 s.f.)

2 **a** ABC is an isosceles triangle

$B = 180° - 2 \times 40°$

$B = 100°$

b $A = \frac{1}{2} \times 10 \times 10 \times \sin 100°$

$A = 49.2$ cm^2

3 **a** $C = 180° - 2 \times 50°$

$C = 80°$

b $A = \frac{1}{2} \times 3 \times 3 \times \sin 80$

$A = 4.43$ m^2

4 Find first the size of one angle.

$\cos X = \frac{20^2 + 16^2 - 8^2}{2 \times 20 \times 16}$

$\cos X = 0.925$

$X = \cos^{-1}(0.925)$

$X = 22.331...°$

$A = \frac{1}{2} \times 20 \times 16 \times \sin 22.3316...°$

$A = 60.8$ km^2 (3 s.f.)

5 **a** $\frac{10}{\sin 100°} = \frac{5}{\sin Y}$

$\sin Y = \frac{5 \sin 100°}{10}$

$\sin Y = 0.4924...$

$Y = \sin^{-1}(0.4924...)$

$Y = 29.498....°$

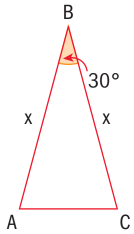
$Z = 180° - 100° - 29.4987...$

$Z = 50.5°$ (3 s.f.)

b $A = \frac{1}{2} \times 50 \times 100 \times \sin 50.5012...°$

$A = 1930$ m^2 (nearest 10 m^2)

6 **a**



$4 = \frac{1}{2} \times x \times x \times \sin 30°$

$4 = \frac{1}{2} \times x^2 \times 0.5$

$4 = 0.25 \times x^2$ or equivalent

b $4 = 0.25 \times x^2$

$x^2 = 16$

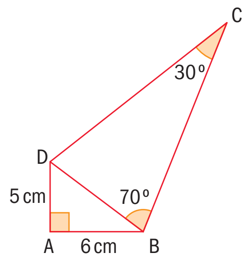
$x = 4$ cm

7 **a** ABD is a right-angled triangle.

$DB^2 = 5^2 + 6^2$

$DB = \sqrt{61}$ cm or 7.81 cm (3 s.f.)

b in triangle BCD



$\frac{\sqrt{61}}{\sin 30°} = \frac{DC}{\sin 70°}$

$DC = \frac{\sqrt{61} \sin 70°}{\sin 30°}$

$DC = 14.7$ cm (3 s.f.)

c from parts **a** and **b**.

BDC = 80^0

$A = \frac{1}{2} \times \sqrt{61} \times 14.678.. \times \sin 80°$

$A = 56.5$ cm^2 (3 s.f.)

d Area of ABCD = Area of ABD + Area of BCD

Area of ABCD = $\frac{1}{2} \times 6 \times 5 + 56.450...$

Area of ABCD = 71.5 cm^2 (3 s.f.)

Review exercise

Paper 1 style questions

1 **a** A(1, 3) and B(5, 1)

$m = \frac{1-3}{5-1}$

$m = -\frac{1}{2}$

b parallel lines have the same gradient. $y = -\frac{1}{2}x + c$

L_2 passes through (0, 4)

$y = -\frac{1}{2}x + 4$ or equivalent forms.

2 **a** use the gradient formula

A(0, 6) and B(6, 0)

$m = \frac{0-6}{6-0}$

$m = -1$

b perpendicular lines have gradients that are opposite and reciprocal.

$m_{\perp} \times m = -1$

$m_{\perp} = \frac{-1}{m}$

$m_{\perp} = \frac{-1}{-1}$

$m_{\perp} = 1$

c $y = 1x + c$

L_2 passes through O (0, 0)

$y = 1x + 0$

$y = x$

3 a i A line meets the x-axis at the point where

$y = 0$

$y = 2x + 3$

$0 = 2x + 3$

$x = -\frac{3}{2}$ (or -1.5)

Point is $\left(\frac{-3}{2}, 0\right)$

ii A line meets the y-axis at the point where

$x = 0$

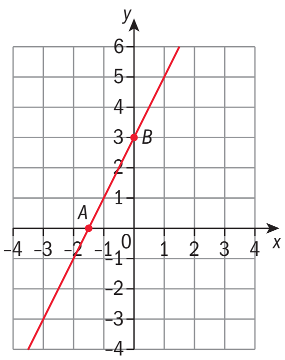
$y = 2x + 3$

$y = 2 \times 0 + 3$

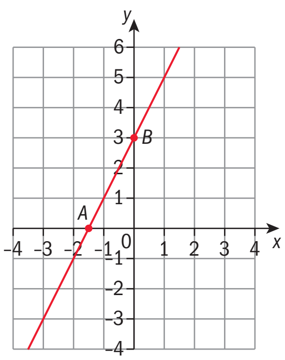
$y = 3$

Point is (0, 3)

b Use the two points found in **a** and draw the line.



c



$\tan\alpha = \frac{3}{1.5}$

$\tan\alpha = 2$

$\alpha = \tan^{-1} 2$

$\alpha = 63.4°$ (3 s.f.)

4 If a point lies on a line then its coordinates verify the equation of the line.

a $y = -2x + 6$

$(a, 4)$ lies on L_1

$4 = -2a + 6$

$a = 1$

b $y = -2x + 6$

$(12.5, b)$ lies on L_1

$b = -2 \times 12.5 + 6$

$b = -19$

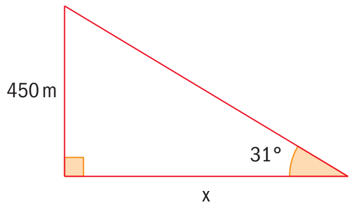
c use the GDC.

$3x - y + 1 = 0$

$y = 3x + 1$ and $y = -2x + 6$

The point is (1, 4)

5 a



b $\tan 31° = \frac{450}{x}$

$x = \frac{450}{\tan 31°}$

$x = 749$ m (3 s.f.)

6 a Sum of the interior angles of a triangle is 180°.

$2 \times 32° + CAB = 180°$

$CAB = 116°$

b $\frac{AB}{\sin 32°} = \frac{20}{\sin 116°}$

$AB = \frac{20 \sin 32°}{\sin 116°}$

$AB = 11.8$ cm (3 s.f.)

c $A = \frac{1}{2} \times 20 \times 11.791\ldots \times \sin 32°$

$A = 62.5\ \text{cm}^2$ (3 s.f.)

7 a AC = 20 − 5 − 6

= 9 m

b Using the cosine rule,

$\cos BAC = \frac{5^2 + 9^2 - 6^2}{2 \times 5 \times 9}$

$BAC = \cos^{-1}(0.777\ldots)$

$BAC = 38.9°$ (3 s.f.)

c $A = \frac{1}{2} \times 5 \times 9 \times \sin 38.9$

$= 14.1\ \text{m}^2$

8 a AO = OB = $\frac{10}{2}$ = 5 cm

$\cos AOB = \frac{5^2 + 5^2 - 7.5^2}{2 \times 5 \times 5}$

$\cos AOB = -0.125$

$AOB = \cos^{-1}(-0.125)$

$AOB = 97.2°$

b $A = \frac{1}{2} \times 5 \times 5 \times \sin 97.180\ldots°$

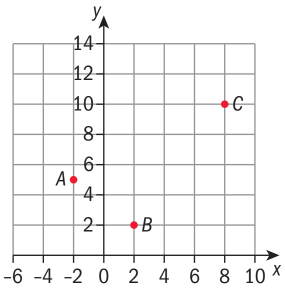
$A = 12.4\ \text{cm}^2$ (3 s.f.)

c Shaded area = $\pi 5^2 - 12.401\ldots$

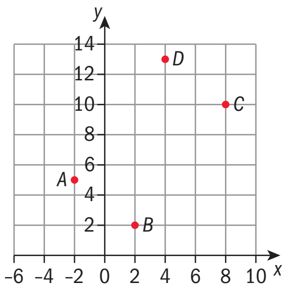
Shaded area = $66.1\ \text{cm}^2$ (3 s.f.)

Review exercise
Paper 2 style questions

1 a



b i



ii D(4,13)

c B(2, 2) and C(8, 10)

$m = \frac{10-2}{8-2}$

$m = \frac{8}{6}$ or $\frac{4}{3}$

d DC and BC are perpendicular lines.

$m_{\perp} \times m = -1$

$m_{\perp} \times \frac{4}{3} = -1$

$m_{\perp} = -\frac{3}{4}$

e use the gradient formula $-\frac{3}{4} = \frac{y-10}{x-8}$

$3(x - 8) = -4(y - 10)$

$3x + 4y - 64 = 0$

f i C(8, 10) and D(4, 13)

$d = \sqrt{(4-8)^2 + (13-10)^2}$

$d = 5$

ii B(2, 2) and C(8, 10)

$d = \sqrt{(8-2)^2 + (10-2)^2}$

$d = 10$

g $\tan DBC = \frac{5}{10}$

$DBC = \tan^{-1}\left(\frac{5}{10}\right)$

$DBC = 26.6°$ (3 s.f.)

2 a Let x be the length of the ladder.

$\cos 60° = \frac{2}{x}$

$x = \frac{2}{\cos 60°}$

$x = 4$ m

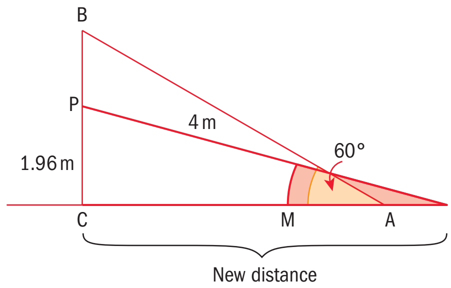
b Let y be the height of the pole.

$\tan 60° = \frac{y}{2}$

$y = 3.46$ m (3 s.f.)

c $3.4641... - 1.5 = 1.96$ m (3 s.f.)

d The length of the ladder is still the same.



Let the new distance be d

$d^2 + 1.9641...^2 = 4^2$

$d = 3.48$ (3 s.f.)

e $\tan \beta = \frac{1.9641...}{3.4845...}$

$\beta = \tan^{-3}\left(\frac{1.9641...}{3.4845...}\right)$

$\beta = 29.4°$ (3 s.f.)

3 a in triangle BCD, $BD^2 = 300^2 + 400^2$

$BD = 500$ m

b in triangle BCD, $\tan BDC = \frac{300}{400}$

$BDC = \tan^{-1}\left(\frac{300}{400}\right)$

$BDC = 36.87°$ (2 d.p.)

c angle ADC = 108°

ADB = 108° – 36.87°= 71.1° (3 s.f.)

d In triangle ADB.

$AB^2 = 500^2 + 1200^2 - 2 \times 500 \times 1200 \times \cos 71.1°$

$AB = 1140$ m (3 s.f.)

e i Perimeter = 1200 + 400 + 300 + 1141.00...

Perimeter = 3040 m (3 s.f.)

ii velocity = $\frac{\text{distance}}{\text{time}}$

$3.8 = \frac{3040}{\text{time}}$

$\text{time} = \frac{3040}{3.8}$

time = 800 seconds

$\text{time} = \frac{800}{60}$ minutes = 13 minutes (nearest minute)

f split the quadrilateral in two triangles

Area ABCD = Area ADB + Area BDC

Area ABCD =

$\frac{1}{2} \times 1200 \times 500 \times \sin 71.1° + \frac{1}{2} \times 400 \times 300$

Area ABCD = 343825 m^2 = 343825 × 10^{-6} km^2 = 0.344 km^2.

4 Mathematical models

Answers

Skills check

1 **a** $y = 2.5x^2 + x - 1$
when $x = -3$, $y = 18.5$

b $h = 3 \times 2^t - 1$ when $t = 0$, $h = 2$

c $d = 2\,t^3 - 5t^{-1} + 2$ when $t = \frac{1}{2}$, $d = \frac{-31}{4}$

2 **a** $x^2 + x - 3 = 0$ $x = 1.30, -2.30$

b $2t^2 - t = 2$
$2t^2 - t - 2 = 0$ $t = -0.781, 1.28$

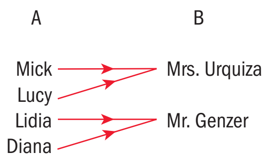
c $x - 2y = 3$
$3x - 5y = -2$ $x = -19, y = -11$

3 **a** A(7, −2) B(−1, 4) $m = \frac{4+2}{-1-7} = \frac{-6}{8} = \frac{-3}{4}$

b A(−3, −2) B(1, 8) $m = \frac{8+2}{1+3} = \frac{10}{4} = \frac{5}{2}$

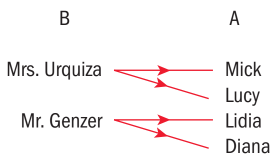
Exercise 4A

1 **a**



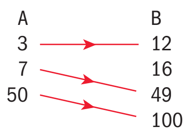
This is a function since each student is in only one mathematics class.

b



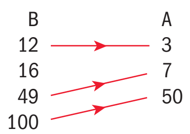
This is not a function since each teachers teaches two of the student.

2 **a**



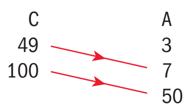
This is a function since each element of A is related to one and only one element of B.

b



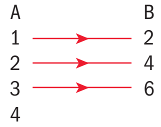
This is not a function since one element of B(16) is not related to any element of A.

c



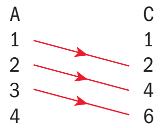
This is a function since each element of C is related to one and only one element of A.

3 **a** **i**



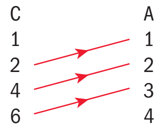
This is not a function since one element of A(4) is not related to any element of B.

ii



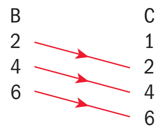
This is not a function since one element of A(4) is not related to any element of C.

iii



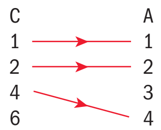
This is not a function since one element of C(1) is not related to any element of A.

iv



This is a function since each element of B is related to one and only one element of C.

v



This is a not a function since one element of C(6) is not related to any element of A.

4 **a** $y = 2x$ **b** $y = \frac{x}{2}$

c $y = \sqrt[3]{x}$ **d** $y = \frac{x^3}{2}$

5 **a** Function

b Function

c not a function since negative elements in the first set are not related to any element in the second set

d Function

Exercise 4B

1 **a** **i**

x	$\frac{-1}{2}$	0	1	3.5	6
$y = 2x$	−1	0	2	7	12

ii domain is the set of all real numbers

iii yes, since $y = 0$ is the image of $x = 0$

b **i**

x	−3	0	2	$\frac{1}{4}$	−2	x
$y = x^2 + 1$	10	1	5	$\frac{17}{16}$	5	5

ii domain is the set of all real numbers

iii no, since there is no solution to the equation

$0 = x^2 + 1$

c **i**

x	−2	−1	0	$\frac{1}{2}$	3	5
$y = \frac{1}{x+1}$	−1	x	1	$\frac{2}{3}$	$\frac{1}{4}$	$\frac{1}{6}$

ii domain is the set of all real numbers except $x = -1$

iii no, since there is no solution to the equation

$0 = \frac{1}{x-1}$

d **i**

x	−3	0	$\frac{1}{4}$	1	9	100
$y = \sqrt{x}$	x	0	$\frac{1}{2}$	1	3	10

ii domain is the set of all non-negative real numbers

iii yes, $y = 0$ is the image of $x = 0$

2 **a** False, there is no solution to the equation $0 = \frac{2}{x}$

b true, $y = x^2 \geq 0$ for all values of x

c true, $y = x^2 + 3 \geq 3$ for all values of x

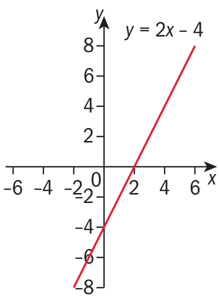
d true, $y = 3$ when $x = \pm 2$

e true, $y = \frac{-3}{3} - 1 = -2$

f false, the image of $x = -1$ is $y = 4$

Exercise 4C

1 **a**



b **i** (2, 0)

ii (0, −4)

c A (250, 490) $490 \neq 2 \times 250 - 4$ no

d B (−3, y) $y = 2 \times -3 - 4 = -10$

2 **a** **i** $\{x: -4 \leq x \leq 6\}$ **ii** $\{y: -4 \leq y \leq 1\}$

iii (4, 0) **iv** (0, −2)

b **i** $\{x: x \in \mathbb{R}\}$ **ii** $\{y: y \leq 8\}$

iii (−4, 0), (0, 0) **iv** (0, 0)

c **i** $\{x: -1 \leq x \leq 1\}$ **ii** $\{y: 0 \leq y \leq 1\}$

iii (−1, 0), (1, 0) **iv** (0, 1)

d **i** $\{x : x \geq -1\}$ **ii** $\{y : y \geq 4\}$

iii no points **iv** (0, 8)

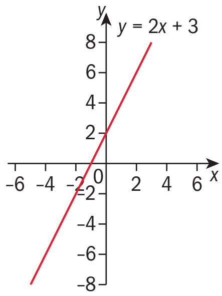
3 **a** **i** F **ii** F **iii** T

b **i** F **ii** T **iii** F

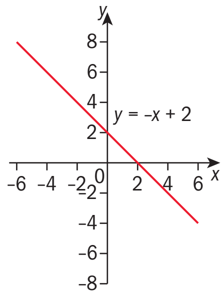
c **i** F **ii** T **iii** F

d **i** F **ii** F **iii** T

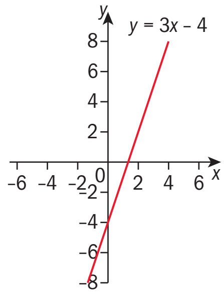
4 **a**



b



c



Exercise 4D

1 $f(x) = x(x-1)(x+3)$

a $f(2) = 2(1)(5) = 10$

b $f\left(\frac{1}{2}\right) = \frac{1}{2}\left(\frac{-1}{2}\right)\left(\frac{7}{2}\right) = \frac{-7}{8}$

c $f(-3) = -3(-4)(0) = 0$

d $f(-1) = -1(-2)(2) = -4$

$\therefore$ (−1, −4) lies on the graph of f

2 $d(t) = 5t - t^2$

a t

b $d(2.5) = 5(2.5) - 2.5^2 = 6.25$

c $d(1) = 5 - 1 = 4$

d $d(1) = 4$ $d(4) = 20 - 16 = 4$ $\therefore$ $d(1) = d(4)$

3 $C(n) = 100 - 10n$

a $C(2) = 100 - 20 = 80$

b $b = C(3) = 100 - 30 = 70$

c $C(a) = 0$ $\therefore$ $100 - 10a = 0$ $\therefore$ $a = 10$

4 **a** **i** $v(1) = 3$ **ii** $v(3) = -3$

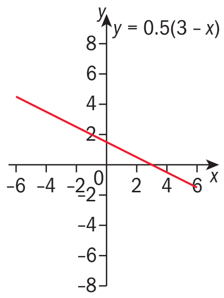
b $-3m + 6 = 9$ $\therefore$ $m = -1$

c $t = 2$

d $v(t) < 0$ for $t > 2$

5 $f(x) = 0.5\,(3 - x)$

a



b (3, 0)

c (0, 1.5)

d $0.5\,(3 - x) = 2 \therefore 3 - x = 4 \therefore x = -1$

6 $h(x) = 3 \times 2^{-2x}$

a **i** $h(0) = 3$ **ii** $h(-1) = 3 \times 2^1 = 6$

b $3 \times 2^{-x} = 24 \quad \therefore 2^{-x} = 8 \quad \therefore x = -3$

Exercise 4E

1 **a** **i** $l = 30 - 2x$ **ii** $w = 15 - 2x$

b $V = (30 - 2x)(15 - 2x)x$

i $V(3)$ is the volume of the box when the squares cut from each corner have side length 3 cm.

ii $V(3) = (24)\,(9)\,(3) = 643\text{ cm}^3$

iii $V(3.4) = (23.2)\,(8.2)\,3.4 = 646.816\text{ cm}^2$

iv No, $x < 7.5$ since the width of the card is only 15 cm

2 **a** width $= 12 - x$

b $A = x\,(12 - x)$

c **i** $A(2)$ is the area of the rectangle when the length is 2 cm

ii $A(2) = 2(10) = 20\text{ cm}^2$

d No, if $x = 12$ the width would be 0.

3 **a** $C = 300 + 150n$

b $C(30) = 300 + 150(30) = 4800$USD

c **i** $300 + 150n \le 2300$

ii $300 + 150(14) = 2400$, no

iii $150n \le 2000$, $n \le 13.3$ 13 days.

4 $C(x) = 0.4x^2 + 1500 \quad \mathrm{I}(x) = -0.6x^2 + 160x$

a $\mathrm{P}(x) = \mathrm{I}(x) - \mathrm{C}(x) = -0.6x^2 + 160x - (0.4x^2 + 1500) = -x^2 + 160x - 1500$

b $\mathrm{P}(6) = -6^2 + 160(6) - 1500 = -576$ AUD, a loss of 576 AUD

c **i** $\mathrm{P}(40) = -40^2 + 160(40) - 1500 = 3300$ AUD

ii $\mathrm{I}(40) = -0.6(40)^2 + 160(40)$
$= 5540$

Assuming all books have same price, one book costs

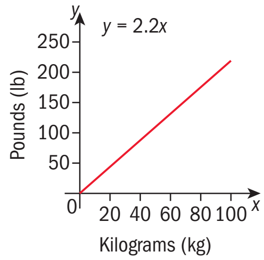
$$\frac{\mathrm{I}(40)}{40} = \frac{5540}{40} = 136\text{ AUD}$$

d 10 or 150

Exercise 4F

1 **a** 50 kg = 110 pounds

b



c gradient = 2.2 $\quad p(x) = 2.2x$

d $p(75) = 165 \quad p(125) = 275$

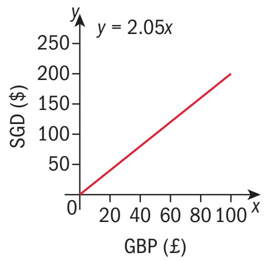
e $y = 2.2x \quad \therefore x = \frac{y}{2.2} \quad k(x) = \frac{x}{2.2}$

f $k(75) = 34.1 \quad k(100) = 45.5$

2 €1 = S$2.05

a €50 = S$102.5

b



c gradient = 2.05 $\quad s(x) = 2.05x$

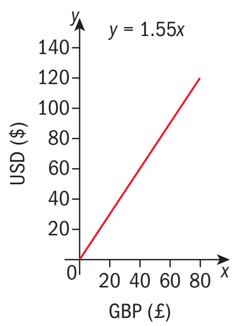
d $s(80) = 164 \quad s(140) = 287$

e $p(x) = \frac{x}{2.05} \quad p(180) = 87.8$

3 €1 = $1.55

a 60 GBP = 93 USD

b



c gradient = 1.55 $\quad u(x) = 1.55x$

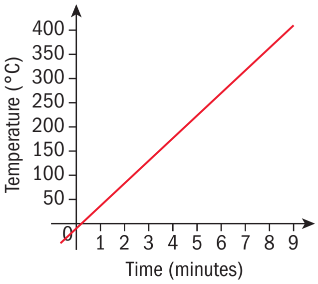
d $u(300) = 465 \quad u(184) = 285.2$

e $p(x) = \frac{x}{1.55}$

f $p(250) = 161 \quad p(7750) = 5000$

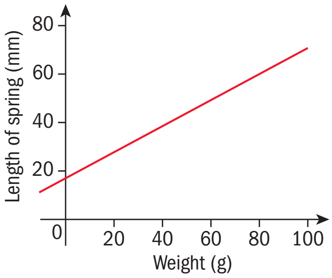
Exercise 4G

1 **a**



b 10°C **c** $T(x) = 40x + 10$

2 **a**



b 18 mm **c** 20 g

d 0.5 mm **e** $L(x) = 0.5x + 18$

3 **b** $T(x) = \frac{2}{3}x + 10$

c 66.7°C

4 **b** 20 cm

c 20 cm

d 350 g

e $L(x) = 0.08x + 20$

Exercise 4H

1 **a** Flour = $80s + 60f$

b Fat = $50s + 90f$

c $80s + 60f = 820$

$50s + 90f = 880$

$s = 5$ $f = 7$ 5 sponge cakes, 7 fruit cakes

2 $8t + 3c = 51$

$100t + 30c = 570$

$t = 3$ $c = 9$ 3 tables, 9 chairs

3 $3v + 5c = 59$

$7v + 3c = 70$

$v = 6.65$ $c = 7.81$ 7 vans, 8 cars

4 $80p + 50t = 620$

$10p + 25t = 190$

$p = 4$ $t = 6$ 4 passenger planes, 6 transport planes

5 $70x + 40y = 1440$

$x = 2y$

$140y + 40y = 1440$

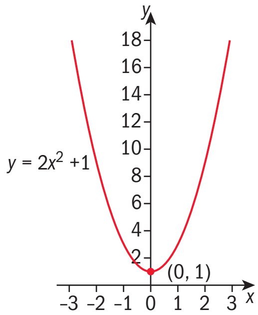
$180y = 1440$

$y = 8$ $x = 16$

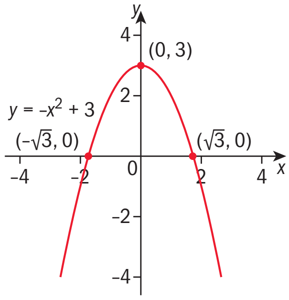
16 volume 1, 8 volume 2

Exercise 4I

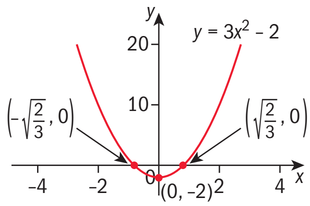
1



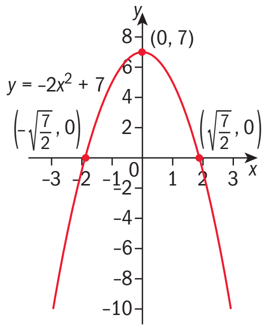
2



3



4



Exercise 4J

1 $(-3, -2)$ $x = -2$

2 $(-5, 4)$ $x = -5$

3 $(4, -1)$ $x = 4$

4 $(5, 7)$ $x = 5$

5 $(-3, 4)$ $x = -3$

Exercise 4K

1 $y = x(x - 4)$

a $x = 2$ **b** $(0, 0)$ $(4, 0)$ **c** $(2, -4)$

2 $y = x(x + 6)$

a $x = -3$ **b** $(0, 0)$ $(-6, 0)$ **c** $(-3, -9)$

3 $y = 8x - x^2 = x(8 - x)$

a $x = 4$ **b** $(0, 0)$ $(8, 0)$ **c** $(4, 16)$

4 $y = 3x - x^2 = x(3 - x)$

a $x = \frac{3}{2}$ **b** $(0, 0)$ $(3, 0)$ **c** $\left(\frac{3}{2}, \frac{9}{4}\right)$

5 $y = x^2 - 2x = x(x - 2)$

a $x = 1$ **b** $(0, 0)\ (2, 0)$ **c** $(1, -1)$

6 $y = x^2 - x = x(x-1)$

a $x = \frac{1}{2}$ **b** $(0, 0)\ (1, 0)$ **c** $\left(\frac{1}{2}, \frac{-1}{4}\right)$

7 $y = x^2 + 4x = x(x + 4)$

a $x = -2$ **b** $(0, 0)\ (-4, 0)$ **c** $(-2, -4)$

8 $y = x^2 + x = x(x + 1)$

a $x = -\frac{1}{2}$ **b** $(0, 0)\ (-1, 0)$ **c** $\left(\frac{-1}{2}, \frac{-1}{4}\right)$

9 $y = (x + 1)(x - 3)$

a $x = 1$ **b** $(-1, 0)\ (3, 0)$ **c** $(1, -4)$

10 $y = (x - 5)(x + 3)$

a $x = 1$ **b** $(5, 0)\ (-3, 0)$ **c** $(1, -16)$

11 $y = (x - 2)(x - 6)$

a $x = 4$ **b** $(2, 0)\ (6, 0)$ **c** $(4, -4)$

12 $y = (x + 2)(x - 4)$

a $x = 1$ **b** $(-2, 0)\ (4, 0)$ **c** $(1, -9)$

Exercise 4L

1 $y = x^2 - 2x + 3$

a $x = 1$ **b** no points **c** $(1, 2)$

2 $y = x^2 + 4x - 5 = (x + 5)(x - 1)$

a $x = -2$ **b** $(-5, 0), (1, 0)$ **c** $(-2, -9)$

3 $y = x^2 + 6x + 4$

a $x = -3$ **b** $(-0.764, 0), (-5.24, 0)$

c $(-3, -5)$

4 $y = 3x^2 - 6x + 2$

a $x = 1$ **b** $(0.423, 0), (1.58, 0)$

c $(1, -1)$

5 $y = 2x^2 - 8x - 1$

a $x = 2$ **b** $(-0.121, 0), (4.12, 0)$ **c** $(2, -9)$

6 $y = 2x^2 + 6x - 7$

a $x = -\frac{3}{2}$ **b** $(0.898, 0), (-3.90, 0)$ **c** $\left(\frac{-3}{2}, \frac{-23}{2}\right)$

7 $y = 0.5x^2 - x + 2$

a $x = 1$ **b** no points **c** $\left(1, \frac{3}{2}\right)$

8 $y = 0.5x^2 + 3x - 4$

a $x = -3$ **b** $(1.12, 0), (-7.12, 0)$ **c** $\left(-3, \frac{-17}{2}\right)$

Exercise 4M

1 $f(x) = x^2 + 2x - 3 = (x + 3)(x - 1)$

a **i** $(0, -3)$

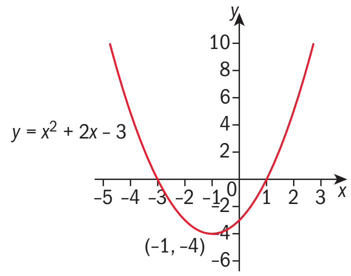
ii $x = -1$

iii $(-1, -4)$

iv $(-3, 0), (1, 0)$

v $y \geq -4$

b



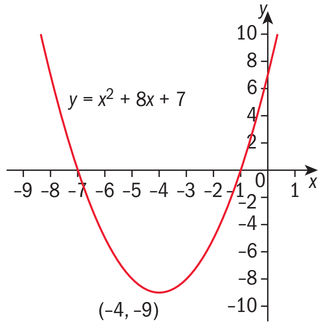
2 $f(x) = x^2 + 8x + 7 = (x + 1)(x + 7)$

a **i** $(0, 7)$ **ii** $x = -4$

iii $(-4, -9)$ **iv** $(-7, 0), (-1, 0)$

v $y \geq -9$

b



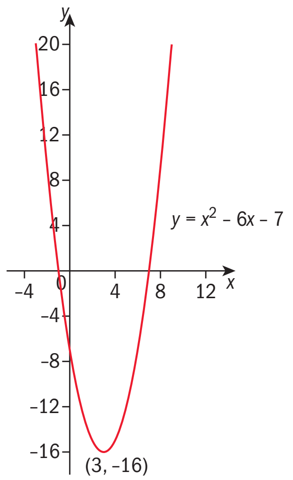
3 $f(x) = x^2 - 6x - 7 = (x - 7)(x + 1)$

a **i** $(0, 7)$ **ii** $x = 3$

iii $(3, -16)$ **iv** $(-1, 0), (7, 0)$

v $y \geq -16$

b



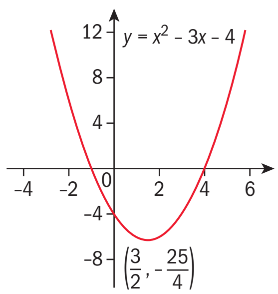
4 $f(x) = x^2 - 3x - 4 = (x - 4)(x + 1)$

a **i** $(0, -4)$ **ii** $x = \frac{3}{2}$

iii $\left(\frac{3}{2}, \frac{-25}{4}\right)$ **iv** $(-1, 0), (4, 0)$

v $y \geq \frac{-25}{4}$

b



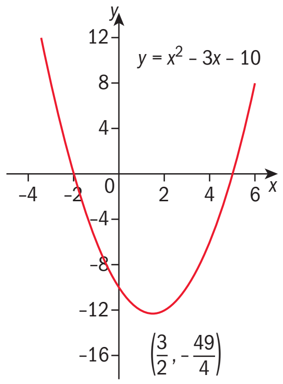
5 $f(x) = x^2 - 3x - 10 = (x - 5)(x + 2)$

a **i** $(0, -10)$ **ii** $x = \frac{3}{2}$

iii $\left(\frac{3}{2}, \frac{-49}{4}\right)$ **iv** $(-2, 0), (5, 0)$

v $y \geq \frac{-49}{4}$

b



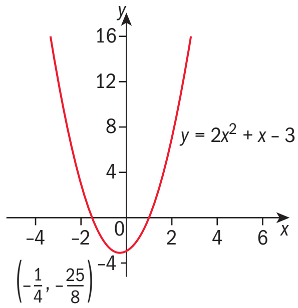
6 $f(x) = 2x^2 + x - 3 = (2x + 3)(x - 1)$

a **i** $(0, -3)$ **ii** $x = \frac{-1}{4}$

iii $\left(\frac{-1}{4}, \frac{-25}{8}\right)$ **iv** $\left(\frac{3}{4}, 0\right), (1, 0)$

v $y \geq \frac{-25}{8}$

b



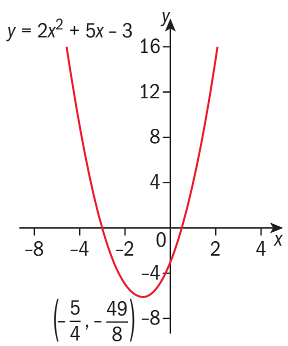
7 $f(x) = 2x^2 + 5x - 3 = (2x - 1)(x + 3)$

a **i** $(0, -3)$ **ii** $x = \frac{-5}{4}$

iii $\left(\frac{-5}{4}, \frac{-49}{8}\right)$ **iv** $\left(\frac{1}{2}, 0\right), (-3, 0)$

v $y \geq \frac{-49}{8}$

b



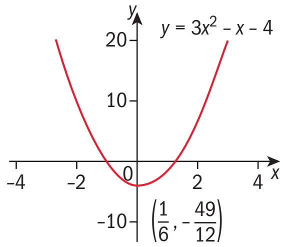
8 $f(x) = 3x^2 - x - 4 = (3x - 4)(x + 1)$

a **i** $(0, -4)$ **ii** $x = \frac{1}{6}$

iii $\left(\frac{1}{6}, \frac{-49}{12}\right)$ **iv** $\left(\frac{3}{4}, 0\right), (-1, 0)$

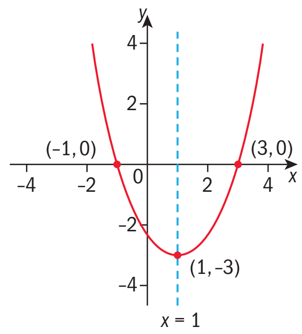
v $y \geq \frac{-49}{12}$

b



Exercise 4N

1



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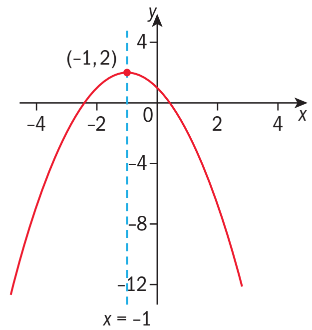

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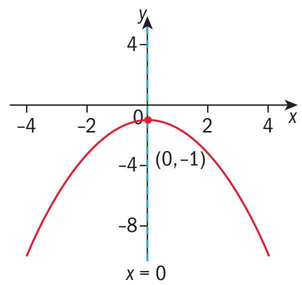

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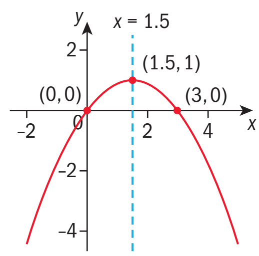

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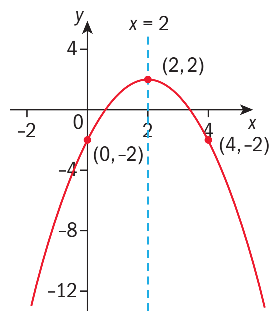
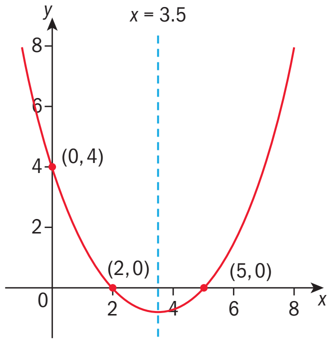


6



Exercise 4O

1 $f(x) = x^2 + 3x - 5 = g(x)\ x - 2$

a $(-3, -5), (1, -1)$

b $x^2 + 3x - 5 = x - 2$

$x^2 + 2x - 3 = 0$

$(x + 3)(x - 1) = 0$

$x = -3$ or 1 (Same as part **a**)

c $h(x) = 2x - 3$

$x^2 + 3x - 5 = 2x - 3$

$x^2 + x - 2 = 0$

$(x + 2)(x - 1) = 0$

$x = -2$ or 1

$(-2, -7), (1, -1)$

2 $f(x) = x^2 + 3x - 5 \quad -5 \le x \le 2$

a $x + y + 5 = 0$

$y = -5 - x$

$x^2 + 3x - 5 = -5 - x$

$x^2 + 4x = 0$

$x(x + 4) = 0$

$x = 0$ or $-4 \quad (0, -5)\ (-4, -1)$

3 a $f(x) = 5 + 3x - x^2 \quad g(x) = 1$

$5 + 3x - x^2 = 1$

$x^2 - 3x - 4 = 0$

$(x - 4)(x + 1) = 0$

$x = 4$ or $-1 \quad (4, 1), (-1, 1)$

b $f(x) = 5 + 3x - x^2 \quad h(x) = 2x + 3$

$5 + 3x - x^2 = 2x + 3$

$x^2 - x - 2 = 0$

$(x - 2)(x + 1) = 0$

$x = 2$ or $-1 \quad (2, 7), (-1, 1)$

4 b f: range $= \{y : -3.125 \le y \le 18\}$

g : range $= \{y : -2 \le y \le 4\}$

c $x = -1$ or 2

e $f(x) = h(x)$

$2x^2 - x - 3 = 2x + 2$

$2x^2 - 3x - 5 = 0$

$(2x - 5)(x + 1) = 0$

$x = \frac{5}{2}$ or -1

f $(-2, 7), (2, 3)$

5 a $(2.12, 1.5), (-2.12, 1.5)$

$f(x) < g(x)$

$-2.12 < x < 2.12$

Exercise 4P

1 $f(x) = ax^2 + bx + c$

$c = -1 \qquad -\frac{b}{2a} = -2 \quad \therefore b = 4a$

$(-2, -5) \qquad -5 = 4a - 2b - 1$

$4a - 2b = -4$

$2a - b = -2$

$2a - 4a = -2$

$-2a = -2$

$a = 1 \quad b = 4$

$f(x) = x^2 + 4x - 1$

$g(x) = ax^2 + bx + c$

$c = -2 \qquad \frac{-b}{2a} = -1 \quad \therefore b = 2a$

$(-1, -3) \qquad -3 = a - b - 2$

$a - b = -1$

$a - 2a = -1$

$a = 1\ b = 2$

$g(x) = x^2 + 2x - 2$

2 $f(x) = ax^2 + bx + 5$

$\frac{-b}{2a} = 2 \quad \therefore b = -4a$

$(2, 1)\ 1 = 4a + 2b + 5$

$4a + 2b = -4$

$2a + b = -2$

$2a - 4a = -2$

$\therefore a = 1, b = -4$

$f(x) = x^2 - 4x + 5$

$g(x) = ax^2 + bx + 3$

$\frac{-b}{2a} = 1 \quad \therefore b = -2a$

$(1, 2) \qquad 2 = a + b + 3$

$a + b = -1$

$a - 2a = -1$

$a + 1 \quad b = -2$

$g(x) = x^2 - 2x + 3$

3 $f(x) = ax^2 + bx + 5$

$\frac{-b}{2a} = 2 \quad \therefore b = -4a$

$(2, 9) \qquad 9 = 4a + 2b + 5$

$2a + b = 2$

$-2a = 2$

$a = -1\ b = 4$

$f(x) = -x^2 + 4x + 5$

$g(x) = ax^2 + bx + 3$

$\frac{-b}{2a} = 1 \quad \therefore b = -2a$

$(1, 4) \qquad 4 = a + b + 3$

$a + b = 1$

$-a = 1$

$a = -1 \quad b = 2 \quad g(x) = -x^2 + 2x + 3$

4 $f(x) = ax^2 + bx + 2$

$\frac{-b}{2a} = -1 \quad \therefore\ b = 2a$

$(-1, 5) \qquad 5 = a - b + 2$

$a - b = 3$

$-a = 3$

$a = -3 \quad b = -6 \quad f(x) = -3x^2 - 6x + 2$

$g(x) = ax^2 + bx - 3$

$\frac{-b}{2a} = -2 \quad \therefore\ b = 4a$

$(-2, 5) \qquad 5 = 4a - 2b - 3$

$2a - b = 4$

$-2a = 4$

$a = -2 \quad b = -8 \quad g(x) = -2x^2 - 8x - 3$

5 $f(x) = ax^2 + bx$

$\frac{-b}{2a} = \frac{-1}{2} \quad \therefore\ b = a$

$\left(\frac{-1}{2}, \frac{-1}{2}\right) \qquad \frac{-1}{2} = \frac{1}{4}a\ \frac{-1}{2}b$

$-2 = a - 2b$

$-2 = -a$

$a = 2 \quad b = 2 \quad f(x) = 2x^2 + 2x$

$g(x) = ax^2 + bx + 3$

$\frac{-b}{2a} = 0 \quad \therefore\ b = 0$

$(1, 2) \qquad 2 = a + 3$

$\therefore\ a = -1 \quad g(x) = -x^2 + 3$

Exercise 4Q

1 **a** $2l + 2w = 170$

$l + w = 85$

$l = 85 - w$

$A = lw = w(85 - w)$

$A = w(85 - w)$

For maximum area, $w = 42.5$, $l = 42.5$

length = 42.5 m, width = 42.5 m

b $2l + w + (w - 15) = 110$

$2l + 2w = 125$

$l = 62.5 - w$

$A = w(62.5 - w)$

For maximum area, $w = 31.25$, $l = 31.25$

length = 31.25 m, width = 31.25 m

2 $p(u) = -0.032u^2 + 46u - 3000$

a 13531.25 riyals

b 3000 riyals

c 69 units

3 $H(t) = 37t - t^2$

a 270 m

b 342.25 m

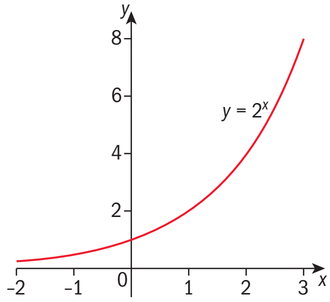
c 37 s

Exercise 4R

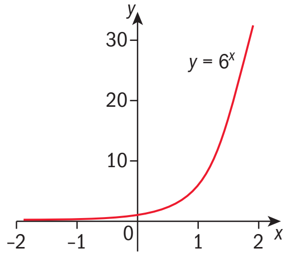
For all questions: y intercept is (0, 1), horizontal asymptote is $y = 0$

Exercise 4S

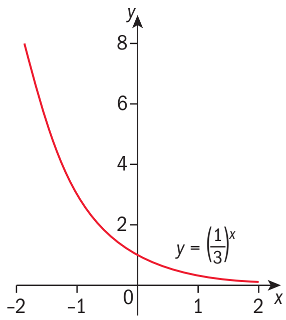
1 $f(x) = 2^x$ **a** (0, 1) **b** $y = 0$



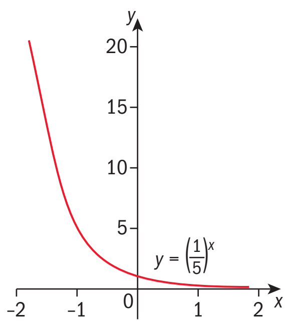
2 $f(x) = 6^x$ **a** (0, 1) **b** $y = 0$



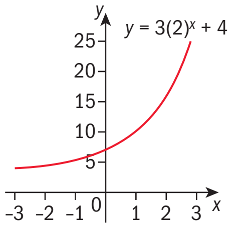
3 $f(x) = \left(\frac{1}{3}\right)^x$ **a** (0, 1) **b** $y = 0$



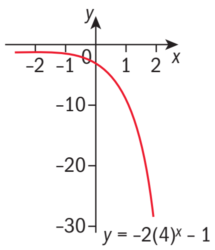
4 $f(x) = \left(\frac{1}{5}\right)^x$ **a** (0, 1) **b** $y = 0$



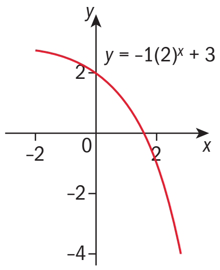
5 $f(x) = 3(2)^x + 4$ **a** (0, 7) **b** $y = 4$



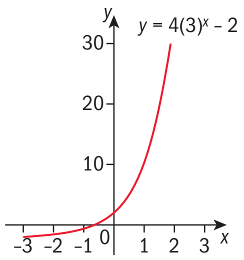
6 $f(x) = -2(4)^x - 1$ **a** (0, −3) **b** $y = -1$



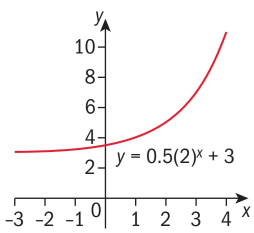
7 $f(x) = -1(2)^x + 3$ **a** (0, 2) **b** $y = 3$



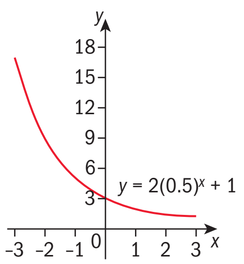
8 $f(x) = 4(3)^x - 2$ **a** (0, 2) **b** $y = -2$



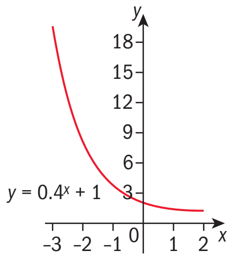
9 $f(x) = 0.5(2)^x + 3$ **a** (0, 3.5) **b** $y = 3$



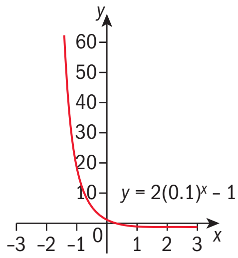
10 $f(x) = 2(0.5)^x + 1$ **a** (0, 3) **b** $y = 1$



11 $f(x) = 0.4^x + 1$ **a** (0, 2) **b** $y = 1$

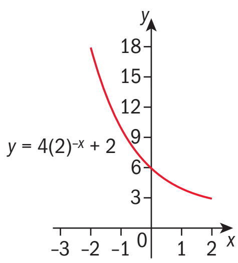


12 $f(x) = 2(0.1)^x - 1$ **a** (0, 1) **b** $y = -1$

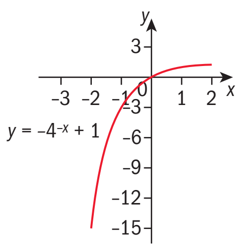


Exercise 4T

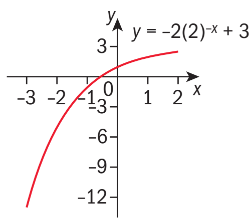
1 $f(x) = 4(2)^{-x} + 2$ **a** (0, 6) **b** $y = 2$



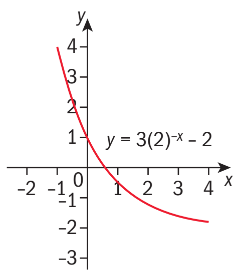
2 $f(x) = -4^{-x} + 1$ **a** (0, 0) **b** $y = 1$



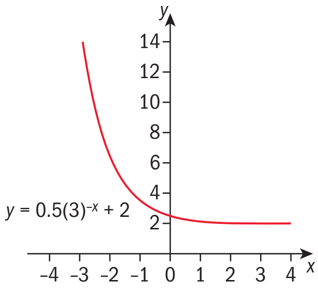
3 $f(x) = -2(2)^{-x} + 3$ **a** (0, 1) **b** $y = 3$



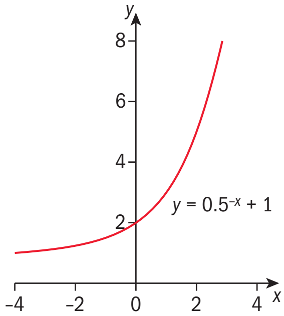
4 $f(x) = 3(2)^{-x} - 2$ **a** (0, 1) **b** $y = -2$



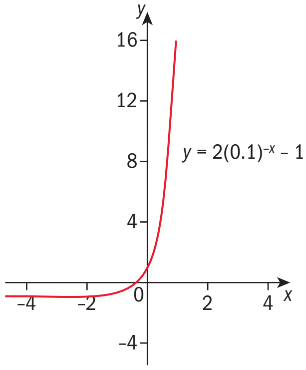
5 $f(x) = 0.5(3)^{-x} + 2$ **a** (0, 2.5) **b** $y = 2$



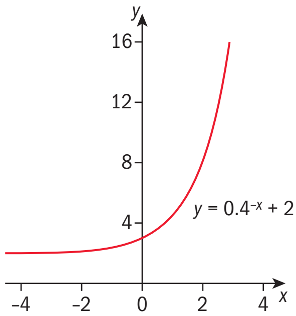
6 $f(x) = 0.5^{-x} + 1$ **a** (0, 2) **b** $y = 1$



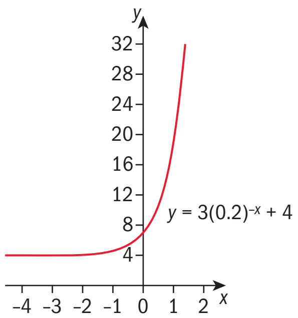
7 $f(x) = 2(0.1)^{-x} - 1$ **a** (0, 1) **b** $y = -1$



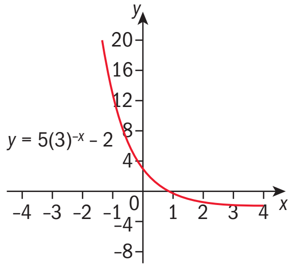
8 $f(x) = 0.4^{-x} + 2$ **a** (0, 3) **b** $y = 2$



9 $f(x) = 3(0.2)^{-x} + 4$ **a** (0, 7) **b** $y = 4$

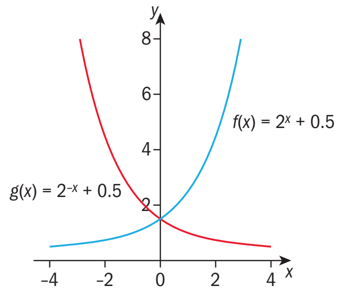


10 $f(x) = 5(3)^{-x} - 2$ **a** (0, 3) **b** $y = -2$



Exercise 4U

1



a (0, 1.5)

b $y = 0.5$

2 $v(t) = 26000\ x^t$

a 26000 euros

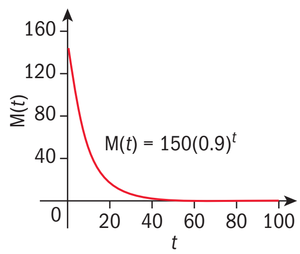
b $26000\ x = 22100$ $\therefore\ x = 0.85$

c $v(t) = 26000\ (0.85)^t$

$v(9) = 6022.04$ $v(10) = 5118.73$ $\therefore$ 10 years

3 $M(t) = 150(0.9)^t$

a



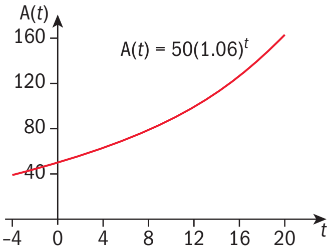
b M = 0

c M(20)= 18.2 g

d M(6) = 79.7 M(7) = 71.7 $\therefore$ 7 years

4 $A(t) = 50(1.06)^t$

a



b days before 1st June.

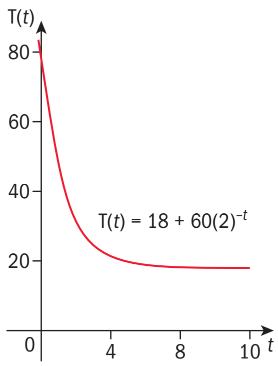
c $A(14) = 113\ \text{m}^2$

d $A(8) = 79.7$ $A(9) = 84.5$ $\therefore\ t = 8$

5 $k + c = -5$ and $0 = 2k + c$, so $c = -10$ $\quad \therefore k = 5$

6 $T(t) = 18 + 60(2)^{-t}$

a



b 78°C

c $T(5) = 19.875$°C

d $T(1.4) = 40.7 \quad T(1.5) = 39.2 \quad \therefore$ 1.5 minutes

e 18°C As t increases T gets closer to 18°C ($T = 18$ is an asymptote).

7 $D(t) = 18000\,(0.9)^t$

a 18000 USD

b $D(5) = 10628.82$ USD

c $D(6) = 9565.94 \quad D(7) = 8609.34 \quad \therefore$ 7 years

8 $f(x) = \frac{2^x}{a} \quad (0, 6) \quad (2, 0.8)$

$b = \frac{1}{a} \quad 0.8 = \frac{4}{a} \quad \therefore a = 5 \quad b = 0.2$

9 $y = 2^x + 3 \quad A(0, a) \quad B(1, b)$

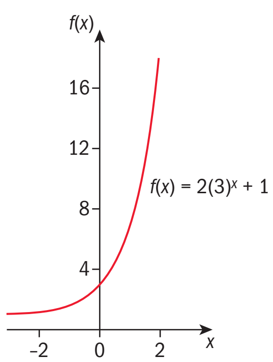
a $a = 2^0 + 3 \quad \therefore a = 4$

$b = 2^1 + 3 \quad \therefore b = 5$

b $y = 3$

10 a $a = 1.667 \quad b = 19$

b



c range $= \{y : y > 1\}$ (or $f(x) > 1$)

Exercise 4V

1 a $f(x) = -0.0015x^4 + 0.056x^3 - 0.60x^2 + 1.65x + 4$

b 8.77 hours

c 1.80 hours, 17.4 hours

2 a 6

b $(x - 2)^4 + 6 = 6$

$(x - 2)^4 = 0$

$x = 2$

c $f(x) \geq 6$

Exercise 4W

1 b 28.9°C

c $50 = 21 + \frac{79}{x}$

$29 = \frac{79}{x} \quad \therefore x = 2.72$ minutes

d $x = 0$

e $y = 21$

f 21°C

2 $f(x) = 100 - \frac{100}{x}$

b 90°C

c $100 - \frac{100}{x} = 30$

$\frac{100}{x} = 70 \quad \therefore x = 1.43$ minutes

d 100°C

3 b $8 = \frac{5}{x^2} \quad x^2 = \frac{5}{8} \quad x = \pm 0.791$

c $x = 0, y = 0$

d $f(x) > 0$

4 b 3.75

c $5 = 3 + \frac{6}{x}$

$\frac{6}{x} = 2 \quad \therefore x = 3$

d $x = 0, y = 3$

e range is all real numbers except 3

$\{y : x \in \mathbb{R}, y \neq 3\}$

Exercise 4X

1 b minimum value = 17.5 (when $x = 1.71$)

c 75.3 ms^{-1}

d $50 = \frac{20}{x} + 2x^2$

$2x^3 - 50x + 20 = 0$

$x = 0.403\,s,\ 4.79\,s$

2 a $v = x(2x)y = 2x^2y$

b $y = \frac{300}{2x^2} = \frac{150}{x^2}$

$A = 2x^2 + xy + xy + 2xy + 2xy$

$A = 2x^2 + 6xy$

$A = 2x^2 + 6x\left(\frac{150}{x^2}\right) = 2x^2 + \frac{900}{x}$

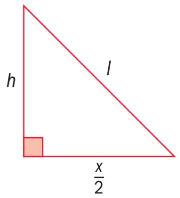
d For minimum area, $x = 6.0822$, $y = \frac{150}{6.0822^2}$

$= 4.0548$

length = 6.08 cm, breadth = 12.2 cm, height = 4.05 cm

3 **a** $v = \frac{1}{3}x^2h$

b



$l^2 = h^2 + \left(\frac{x}{2}\right)^2 \qquad l = \sqrt{h^2 + \left(\frac{x}{2}\right)^2}$

c $A = x^2 + \frac{4xl}{2} = x^2 + 2xl = x^2 + 2x\sqrt{h^2 + \left(\frac{x}{2}\right)^2}$

d $1500 = \frac{x^2h}{3} \quad \therefore\ h = \frac{4500}{x^2}$

$l = \sqrt{\frac{4500^2}{x^4} + \frac{x^2}{4}} \qquad A = x^2 + 2x\sqrt{\frac{4500^2}{x^4} + \frac{x^2}{4}}$

f For minimum area, $x = 14.7084$, $h = \frac{4500}{14.7084^2}$

$= 20.8009$

side length = 14.7 m, height = 20.8 m

4 Let width $= x$, length $= 2x$, height $= h$

$320 = 12x + 4h$

$h = 80 - 3x$

viewing area $= 2xh + 2xh + xh$

$= 5xh$

$A = 5x(80 - 3x)$

maximum viewing area = 2666.67

= 2670 cm^2

Exercise 4Y

1 $f(x) = 1 + \frac{2}{x}, x \neq 0$

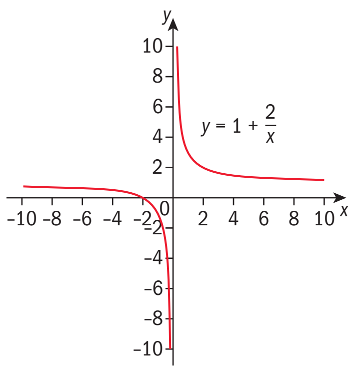
a $\{x : x \in \mathbb{R}, x \neq 0\}$

b

x	−10	−5	−4	−2	−1	−0.5	−0.2	0
f(x)	0.8	0.6	0.5	0	−1	−3	−9	

x	0.2	0.5	1	2	4	5	10
f(x)	11	5	3	2	1.5	1.4	1.2

c



d $x = 0$

e $y = 1$

2 $f(x) = 8x^{-1} + 3, x \neq 0$

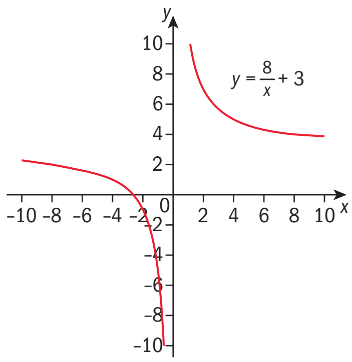
a $\{x : x \in \mathbb{R}, x \neq 0\}$

x	−10	−8	−5	−4	−2	−1	0
f(x)	2.2	2	1.4	1	−1	−5	0

b

x	1	2	4	5	8	10
f(x)	11	7	5	4.6	4	3.8

c



d $x = 0$

e $y = 3$

Exercise 4Z

Sketch graphs

1 Range : $y \geq 1.81$

2 $y \in \mathbb{R}$

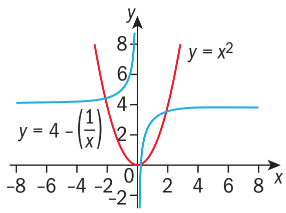
3 $y \in \mathbb{R}$

4 $y \geq -1.25$

5 $y < 0$ or $y \geq 2.98$

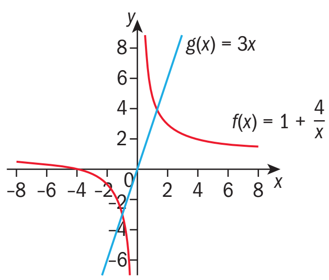
Exercise 4AA

1 **a**



b (0.254, 0.0646), (1.86, 3.46), (−2.11, 4.47)

2 **a, c**



b $y = 1, x = 0$

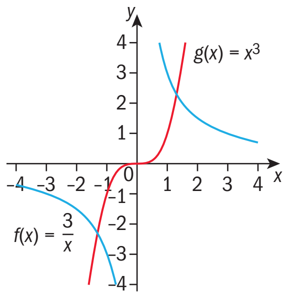
d $1 + \frac{4}{x} = 3x \quad x = -1$ or 1.33

e $\{y : y \in \mathbb{R}, y \neq 1\}$

3 **a** $(-0.366, 0.669), (0.633, 2.01)$

b $y = 0$

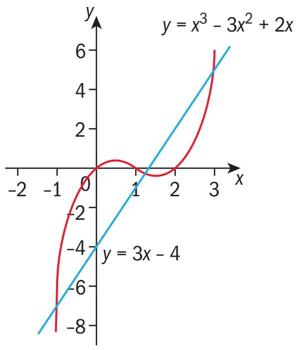
4 **a**



b $\frac{3}{x} - x^3 = 0 \quad \frac{3}{x} = x^3 \quad$ 2 solutions

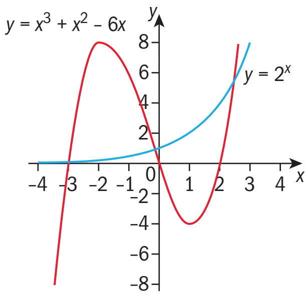
c 1.32 or −1.32

5



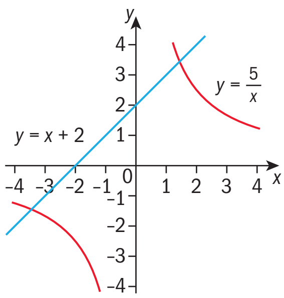
$(-1.11, -7.34), (1.25, -0.238), (2.86, 4.58)$

6



$(-2.99, 0.126), (-0.147, 0.903), (2.41, 5.31)$

7



a $x = 1.45$ or -3.45

b $y = 0$

c $x = 0$

Exercise 4AB

1 **a** time in hours, water consumption in litres

b 0700 – 2000

c 0700 – 1200, 1400 – 1600

d 1200 – 1400, 1600 – 2000

e 1200 (local maximum at 1600)

f 0700, 2000 (local minimum at 1400)

2 **a** time in minutes, temperature in °C

b 100°C

c 35°C

d $\frac{1}{2}$ minute

e no

f approximately 22°C

3 **a**

t	0	5	10	15	20
N	1	2	4	8	16

b $13\,s$

c $2^{(60 \div 5)} = 2^{12} = 4096$

4 **a** 45 m

b 1.5 s and 5.5 s

c 0 – 3.5 s

d 3.5 s – 7 s

e 90 m, 3.5 s

f ball returns to ground level

5 **a** **i** 3.8 m **ii** 2.2 m **iii** 0200 and 0600

b $2 < t < 6$

6 **a** twice

b 0400 – 0900

c 1600

d 5°C

e 1100 – 1600

f 1300 and 1930

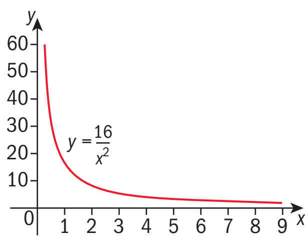
g no, the temperature at the start of the following day is 1°C whereas it was 3°C at the start of this day.

7 **a** $x^2y = 16 \quad \therefore\ y = \frac{16}{x^2}$

b

x	0.5	1	2	4	8	10
***y* = *f*(*x*)**	64	16	4	1	0.25	0.16

c



d tends to zero

8 **a** 3000 cm^3

b $x^2y = 3000 \quad \therefore \quad y = \frac{3000}{x^2}$

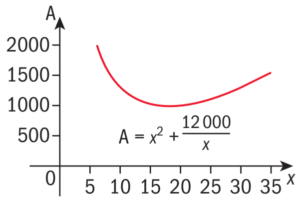
c $A = x^2 + 4xy = x^2 + 4x\left(\frac{3000}{x^2}\right)$

$A = x^2 + \frac{12000}{x}$

d

x (m)	5	10	15	20	25	30	35
A(x) (cm²) (2sf)	2400	1300	1000	1000	1100	1300	1600

e



f $x = 18.2$

Review exercise

Paper 1 style questions

1 **a** 0000 – 0600

b 1130 – 1700

c 13° C

2 $c = nr + s$

a $35000 = 6r + s$

$116000 = 24r + s$

$18r = 81000$

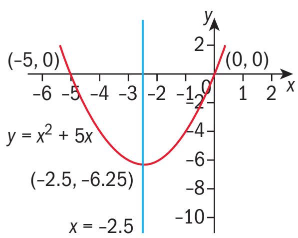
$\therefore\ r = 4500$ SGD

b $35000 = 6 \times 4500 + s$

$s = 8000$ SGD

3 **a** $x^2 + 5x = x(x + 5)$

b



4 $h(t) = 30t - 5t^2 \qquad 0 \le t \le 6$

a $h(4) = 40$ m

b 45 m

c from $t = 1$ to $t = 5$, $\therefore$ 4 s

5 **a** $f(x) = \frac{2^x}{m}$

$(3, 1.6) \quad 1.6 = \frac{2^3}{m} \qquad \therefore\ m = \frac{8}{1.6} = 5$

b $f(x) = \frac{2^x}{5}$

$(0, n) \quad n = \frac{1}{5}$

$f(2) = \frac{2^2}{5} = \frac{4}{5}$

6 **a** $x^2 - 2x - 15 = (x - 5)(x + 3)$

b **i** At A, $x = -3$ $\qquad$ A = (−3, 0)

ii At B, $x = 1$ $\qquad$ B = (1, −16)

7 **a** **ii**

b **i**

c **iii**

d **iv**

8 **a** **i** $A(-1.68, 1.19)$

ii $B(2.41, -1.81)$

b $f(x) < g(x) \quad -1.68 < x < 2.41$

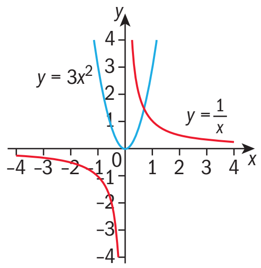
c $y = -2$

9 **a** width $= 2.2 - x$

b $A = x(2.2 - x)$

c For maximum area, $x = 1.1$ m

10 **a**



b $x = 0,\ y = 0$

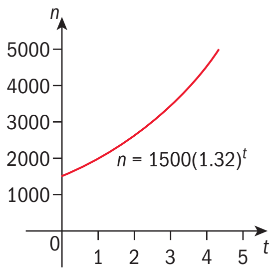
c $x = 0.693$

Paper 2 style questions

1 $n = 1500(1.32)^t$

a 1980, 4554

b

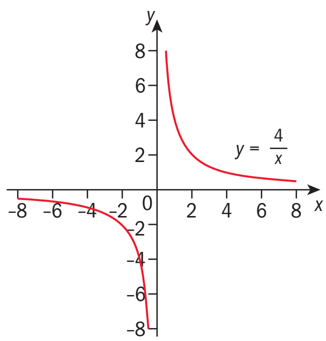


c **i** $n = 1500(1.32)^{2.5} = 3000$

ii $t = 4.3366$ hours

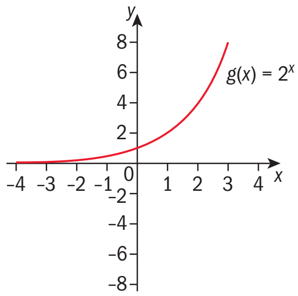
$= 4$ hours 20 mins

2 **a**



b $y = 0, x = 0$

c



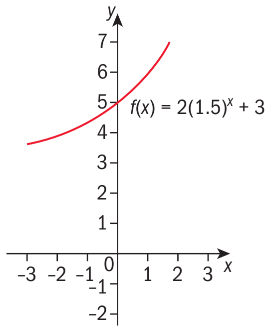
d $x = \pm 1.41$

e $\{y : y \in \mathbb{R}, y \neq 0\}$

3 $f(x) = 2(1.5)^x + 3$

a $a = 4.33 \quad b = 7.5$

b



c $f(x) > 3$

d $x = 3.09$

e $y = 3$

4 $f(t) = 21 + 77(0.8)^t$

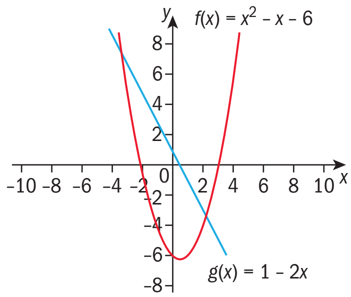
a $f(0) = 21 + 77 = 98°C$

b $y = 21$

c 21°C

d $f(8) = 33.9°C$

5 **a**



b $(0.5, -6.25)$

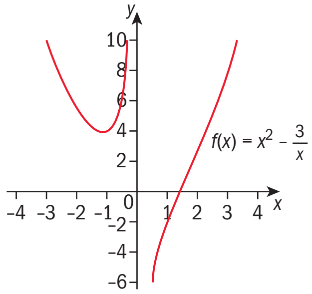
c -2

d $(0, 1)$

e $(2.19, -3.39)$, $(-3.19, 7.39)$

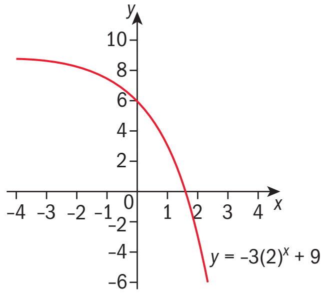
f $x = 2.19, -3.19$

6 **a**



b $x = 0$

c



d $y = 9$

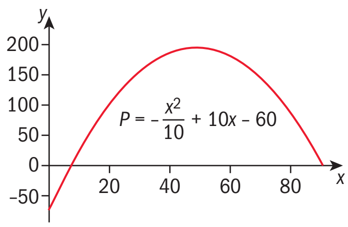
e $(-2.73, 8.55)$, $(-0.454, 6.81)$, $(1.53, 0.362)$

7 $P = \frac{-x^2}{10} + 10x - 60$

a

x	0	10	20	30	40	50	60	70	80	90
P	−60	30	100	150	180	190	180	150	100	30

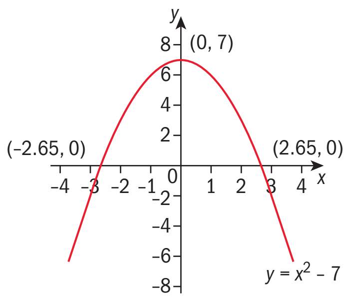
b



c **i** 190 euros **ii** 50 **iii** 33 or 67

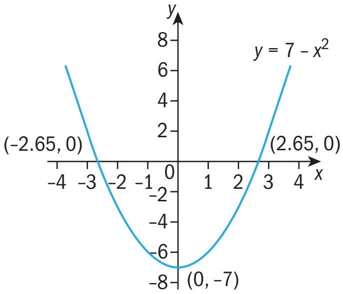
iv 60 euros

8 **a**



$(0, -7)$, $(2.65, 0)$, $(-2.65, 0)$

b



c $x = \pm 2.65$

d $c = 1, 2, 3, 4, 5, 6$

9 **a** $\frac{x^2}{2} = \frac{-x^2}{2} + 2x$

$x^2 - 2x = 0$

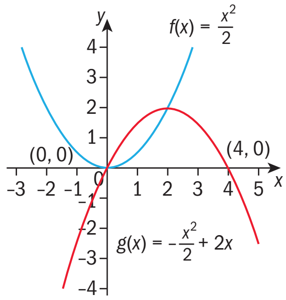
$x(x - 2) = 0$

$x = 0$ or 2 $(0, 0), (2, 2)$

b $x = \frac{-b}{2a} = \frac{-2}{-1} = 2 \quad x = 2$

c $k = 2$

d



e $f(x) < g(x)$

$0 < x < 2$

5 Statistical applications

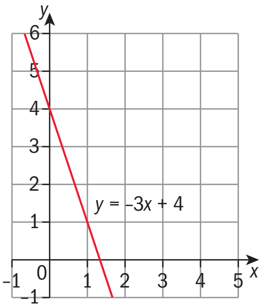
Answers

Skills check

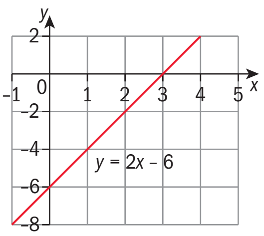
1 **a** mean = 3.61 standard deviation = 1.21
The small standard deviation implies that the data are close to the mean

b mean = 14 standard deviation = 0.643
The mean is the middle data value (14) since the frequencies are symmetrical about this value. The standard deviation is very small since most of the data values equal the mean and the rest are close to it

2 **a**

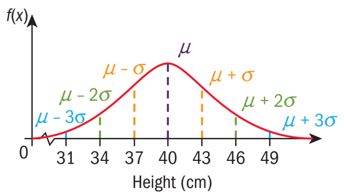


b



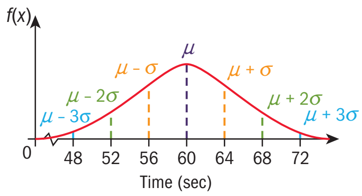
Exercise 5A

1 **a**



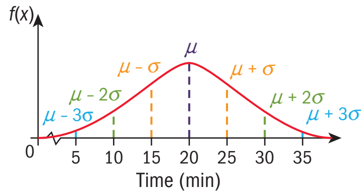
b 0.16 **c** 0.815 **d** 200 × 0.16 = 32

2 **a**



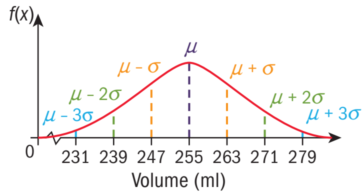
b 81.5% **c** 100 × 0.5 = 50

3 **a**



b 16% **c** 60 × 0.68 = 40.8 or 41

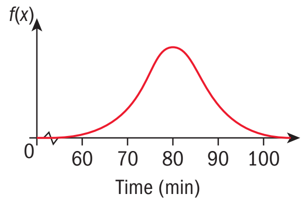
4 **a**



b 0.025 **c** 75 × 0.84 = 63

Exercise 5B

1 **a**



b 0.0766

c 365 × 0.0766 = 27.959 or 28 days

2 **a** **i** P(IQ < 90) = 0.159

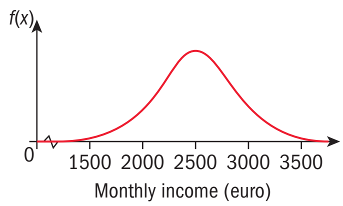
ii P(IQ > 120) = 0.0228

iii P(80 < IQ < 110) = 0.819

b P(IQ > 115) = 0.0668, 2000 × 0.0668 = 134

3 **a** 0.0668 **b** 0.00621 **c** 300 × 0.927 = 278

4 **a**



b 0.0401

c 80 × 0.0122 = 0.976 or 1

5 **a** 78.9%

b 0.00621

c 100 × 0.0304 = 3.04 or 3

6 0.106

7 **a** 86.4%

b 30 × 0.0304 = 0.912 or 1

8 mean = 1.78 m standard deviation = 0.02 m

a 0.00621

b 3

Exercise 5C

1 $p = 4.93$

2 $h = 183$

3 $k = 20.8$

4 $w = 222$

5 **a** 3.47 to 4.99 kg

b $180 \times 0.683 = 123$

c 0.0685

d 87.7%

e $w = 5.48$

6 **a** $a = 29$, $b = 30$, $c = 31$

b 0.919

c $d = 32.8$

d $5000 \times 0.6246....$

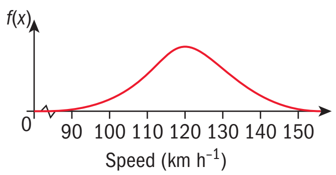
$= 3123$ (accept 3120 to 3125)

7 **a** 0.000429

b 0.854

c $t = 5885$

8 **a**



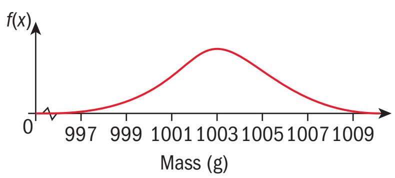
b 62.5%

c $p = 106$

d $800 \times 0.911 = 729$

e $800 \times 0.159 = 127$

9 **a**



b 0.0228 **c** 0.0668

d $400 \times 0.0668 = 26.7$ or 27

e $p = 1006$

10 **a** 0.466%

b A baby weighing 2.34 kg (2.34 is nearer the mean than 5.5).

c $300 \times 0.0808 = 24.2$ or 24

d $w = 3.16$

Exercise 5D

1 **a** strong positive linear

b moderate negative linear

c moderate positive linear

d weak positive linear

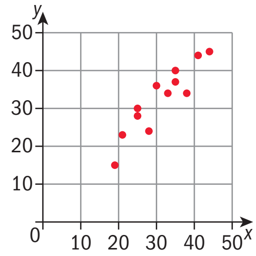
e none

f perfect negative linear

g non-linear

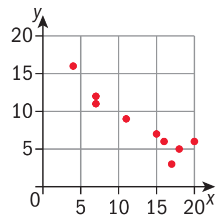
h weak negative linear

2 **a**



moderate positive linear correlation

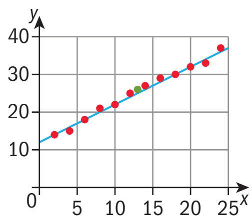
b



moderate negative linear correlation

Exercise 5E

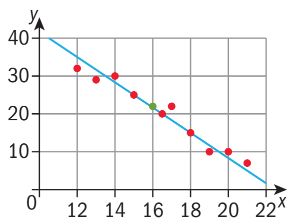
1 **i** **a, c**



strong positive correlation

b mean of $x = 13$, mean of $y = 25.75$

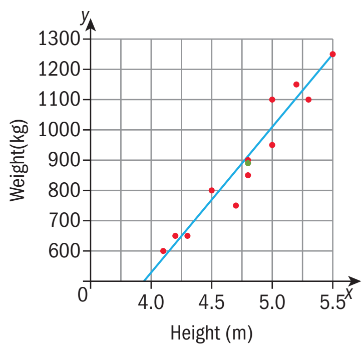
ii **a, c**



strong negative correlation

b mean of $x = 16.5$, mean of $y = 20.2$

2 **a, c**



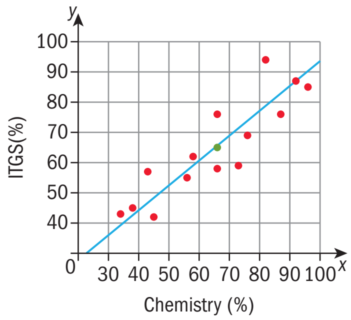
moderate positive correlation

b mean height = 4.78 m

mean weight = 896 kg

d 820 kg

3 a, c

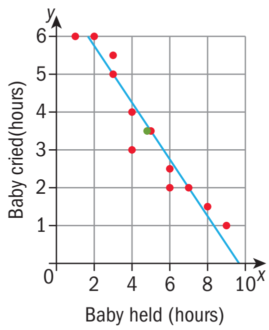


weak positive correlation

b chemistry mean = 65.3
ITGS mean = 65.1

d 52%

4 a, c

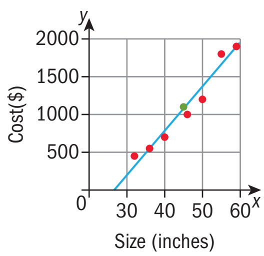


moderate negative correlation

b mean number of hours held = 4.83
mean number of hours spent crying = 3.5

d 4.5 hours

5 a, c



moderate positive correlation

b mean screen size = 45.6 inches
mean cost = \$ 1100

d \$1540

Exercise 5F

1 $r = 0.931$, strong positive correlation

2 a $r = 0.880$

b strong positive correlation

3 $r = -0.891$, strong negative correlation

4 $r = 0.936$, strong positive correlation

5 $r = 0.990$, strong positive correlation

6 $r = 0.200$, very weak positive correlation

7 $r = 0.985$, strong positive correlation

8 $r = 0.580$, moderate positive correlation

Exercise 5G

1 a $r = 0.994$, strong positive correlation

b $y = 1.47x + 116$

c $y = 1.47(1000) + 116 = 1586$, £1590 (3 s.f)

2 a $r = 0.974$

b $y = 0.483x + 15.6$

c $y = 0.483(8) + 15.6 = 19.464$, 19.5 cm

3 a $\bar{x} = 68.6$ $s_x = 6.55$ $\bar{y} = 138$ $s_y = 5.97$

b $r = -0.860$

c strong negative correlation

d $y = -0.784x + 192$

e $y = -0.784(70) + 192 = 137.12$,
137 seconds

4 a $r = 0.792$

b $y = 0.193x + 1.22$

c $y = 0.193(15) + 1.22 = 4.115$, 4.12

5 a $y = 0.0127x + 0.688$

b $y = 0.0127x(70) + 0.688 = 1.577$, 1.58 AUD

6 a $y = 0.751x + 11.6$

b $y = 0.751(50) + 11.6 = 49.15$, 49 situps

7 a $y = 1.04x - 2.53$

b $y = 1.04(60) - 2.53 = 59.87$, 59.9

8 a $y = 0.279x + 2.20$

b $y = 0.279(40) + 2.20 = 13.36$, 13.4 hours.

Exercise 5H

1 a H_0: Genre of book is independent of age
H_1: Genre of book is dependent on age

b $\frac{97}{300} \times \frac{130}{300} \times 300 = 42.0$

c $(3 - 1)(3 - 1) = 4$

d $\chi^2_{calc} = 26.9$

e 26.9 > 9.488, therefore we reject the null hypothesis. There is enough evidence to conclude that genre of book is dependent on age. (p-value = 0.0000207 < 0.05)

2 a H_0: Hair color and eye color are independent
H_1: Hair color and eye color are dependent.

b $\frac{85}{227} \times \frac{90}{227} \times 227 = 33.7$

c $(3 - 1)(3 - 1) = 4$

d $\chi^2_{calc} = 44.3$

e 44.3 > 7.779, therefore we reject the null hypothesis. There is enough evidence to conclude that hair colour and eye color are dependent.
(p-value = 0.00000000556 < 0.1)

3 **a** H_0: Favorite flavor is independent of race.
H_1: Favorite flavor is dependent on race.

b $\frac{35}{140} \times \frac{44}{140} \times 140 = 11$

c $(4 - 1)(3 - 1) = 6$

d $\chi^2_{calc} = 0.675$

e 0.675 < 12.59, therefore we do not reject the null hypothesis. There is enough evidence to conclude that favourite flavor is independent of race. (p-value = 0.995 > 0.05)

4 **a** H_0: Film genre is independent of gender
H_1: Film genre is dependent on gender

b $\frac{39}{80} \times \frac{21}{80} \times 30 = 10.2$

c $(2 - 1)(4 - 1) = 3$

d $\chi^2_{calc} = 19.0$

e 19.0 > 11.345, therefore we reject the null hypothesis. There is enough evidence to conclude that film genre is dependent on gender. (p-value = 0.000276 < 0.01)

5 **a** H_0: Grade is independent of the number of hours
H_1: Grade is dependent on the number of hours

b $\frac{90}{220} \times \frac{96}{220} \times 220 = 39.3$

c $(3 - 1)(3 - 1) = 4$

d $\chi^2_{calc} = 42.1$

e 42.1 > 9.488, therefore we reject the null hypothesis. There is enough evidence to conclude that grade is dependent on number of hours spents playing computer games. (p-value = 0.0000000159 < 0.05)

6 **a** H_0: Employment grade is independent of gender
H_1: Employment grade is dependent on gender

b

	Directors	Management	Teachers
Male	11.5	71.5	538.9
Female	20.5	127.5	960.1

c $(2 - 1)(3 - 1) = 2$

d $\chi^2_{calc} = 180$

e 180 > 4.605, therefore we reject the null hypothesis. There is enough evidence to conclude that employment grade is dependent on grade. (p-value = $8.08 \times 10^{-40} < 0.1$)

7 **a** H_0: Amount of sushi sold is independent the day of the week
H_1: Amount of sushi sold is dependent on the day of the week.

b $\frac{70}{470} \times \frac{145}{470} \times 470 = 52.4$

c $(3 - 1)(3 - 1) = 4$

d $\chi^2_{calc} = 0.840$

e 0.840 < 9.488, therefore we do not reject the null hypothesis. There is enough evidence to conclude that the amount of sushi sold is independent of the day of the week. (p-value 0.933 > 0.05)

8 **a** H_0: A puppy's weight is independent of its parent's weight.
H_1: A puppy's weight is dependent on its parent's weight

b $\frac{46}{141} \times \frac{41}{141} \times 141 = 13.4$

c $(3 - 1)(3 - 1) = 4$

d $\chi^2_{calc} = 13.$

e 13.7 > 13.277, therefore we reject the null hypothesis. There is enough evidence to conclude that a puppy's weight is dependent on its parent's weight.

9 **a** H_0: Music preference is independent of age
H_1: Music preference is dependent on age

b $\frac{137}{419} \times \frac{101}{419} \times 419 = 33.0$

c $(3 - 1)(4 - 1) = 6$

d $\chi^2_{calc} = 31.5$

e 31.5 > 12.59, therefore we reject the null hypothesis. There is enough evidence to conclude that music preference is dependent on age. (p-value = 0.0000204 < 0.05)

10 **a** H_0: Age at which a baby is potty trained is independent of gender.
H_1: Age at which a baby is potty trained is dependent on gender.

b $\frac{140}{300} \times \frac{69}{300} \times 300 = 32.2$

c $(2 - 1)(3 - 1) = 2$

d $\chi^2_{calc} = 51.6$

e 51.6 > 4.605, therefore we reject the null hypothesis. There is enough evidence to conclude that the age at which a baby in potty trained is dependent on gender. ($p = 6.23 \times 10^{-12} < 0.1$).

11 **a** H_0: Grade is independent of gender
H_1: Grade in dependent on gender

b

	5,6 or 7	3 or 4	1 or 2
Male	25.2	66.6	15.1
Female	24.8	65.4	14.9

c $(2 - 1)(3 - 1) = 2$

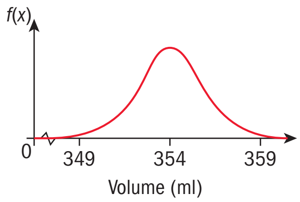
d $\chi^2_{calc} = 0.467$

e $0.467 < 5.991$, therefore we do not reject the null hypothesis. There is enough evidence to conclude that grade is independent of gender. $(p = 0.792 > 0.05)$

Review exercise

Paper 1 style questions

1 **a**



b 0.0548 **c** $100 \times 0.0548 = 5.48$, 5 cans

2 **a** 32.2%

b $6000 \times 0.00982 = 58.9$, 59 people

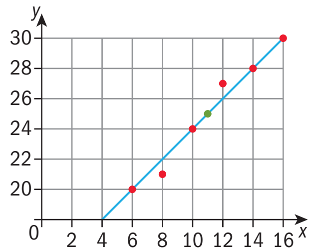
3 **a** 93.3% **b** $p = 1.01$

4 **a** strong positive correlation

b no correlation

c moderate negative correlation

5



a strong positive correlation

b $\bar{x} = 11$ **c** $\bar{y} = 25$

d 23

6 **a** $r = 0.980$, strong positive correlation

b $y = 0.801x - 77.4$

c $y = 0.801(170) - 77.4 = 58.77$, 58.8 cm

7 **a** $r = 0.810$, strong positive correlation

b $y = 0.215x + 14.3$

c $y = 0.215x(40) + 14.3 = 22.9$ seconds

8 H_0: Flavor of ice creams is independent of age
H_1: Flavor of ice creams is dependent on age
Expected values

	$x < 25$	$25 \le x < 45$	$x \ge 45$
Vanilla	14.06	11.84	11.1
Strawberry	10.64	8.96	8.4
Chocolate	13.3	11.2	10.5

degrees of freedom $= (3 - 1)(3 - 1) = 4$
p-value $= 0.963 > 0.05$, $\chi^2_{calc} = 0.604$
We do not reject the null hypothesis. There is enough evidence to conclude that flavor of ice cream is independent of age.
(critical value $= 9.488$, $(\chi^2_{calc} = 0.604 < 9.488)$

9 **a** H_0: The number of pins knocked down is independent of which hand is used.

b $(2 - 1)(3 - 1) = 2$

c $\frac{20}{120} \times \frac{60}{120} \times 120 = 10$

d p-value $= 0.422 > 0.1$ (significance value). Therefore we do not reject the null hypothesis. There is enough evidence to conclude that the number of pins knocked down is independent of which hand is used.

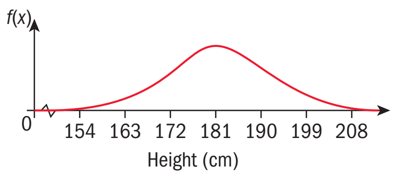
10 **a** H_0: The outcome is independent of the time spent preparing for a test.

b $(3 - 1)(2 - 1) = 2$

c p-value $= 0.069 > 0.05$, therefore we do not reject the null hypothesis. There is enough evidence to conclude that the outcome is independent of the time spent preparing for a test.

Paper 2 style questions

1 **a**



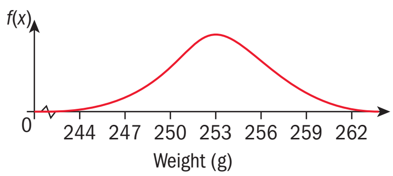
b 0.252

c 0.731

d $60 \times 0.0599 = 3.59$, 4 men

e $k = 166$

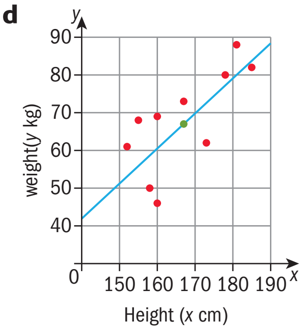
2 **a**



b 0.159

c $300 \times 0.252 = 75.6$, 76 sweets

3 **a, d**



b $\bar{x} = 166.9$

c $\bar{y} = 67.3$

d **i** $y = 0.719x - 52.8$

e 69 kg

4 **a** $r = 0.823$

b strong positive correlation

c $y = 0.219x + 3.85$

d $y = 0.29(35) + 3.85 = 11.515$
$= 12$ hours (nearest hr)

5 **a** $r = 0.866 = 0.9$ (1 d.p.)

b strong positive correlation

c $y = 0.0666x - 2.36$

6 **a** $r = 0.887 = 0.89 = 0.89$ (2 d.p.)

b strong positive correlation

c $y = 0.015x + 0.229$

d $y = 0.0151(80) + 0.229 = 1.437$, 1.44 euros

7 **a** $y = 0.163x - 15.0$

b $y = 0.163\,(170) - 15.0 = 12.71$, dress size 13

c $r = 0.741$

d moderate positive correlation

8 H_0: Choice of game is independent of gender
H_1: Choice of game depends on gender
Expected values:

	Badminton	Table tennis	Darts
Male	39.4	14.8	26.8
Female	29.6	11.2	20.2

degrees of freedom = (2 − 1) (3 − 1) = 2

$\chi^2_{calc} = 0.667$ p-value $= 0.717 > 0.05$

We do not reject the null hypothesis. There is enough evidence to conclude that choice of game is independent of gender.

(critical value = 5.991, $\chi^2_{calc} = 0.667 < 5.991$)

9 **a** $p = 21.6$ $q = 14.4$ $r = 13.6$

b **i** H_0: The extra-curricular activity is independent of gender

ii $(2 - 1)(3 - 1) = 2$

c $\chi^2_{calc} = 4.613$

d $4.613 > 4.605$, therefore we reject the null hypothesis. There is enough evidence to conclude that extra-curricular activity is dependent on gender.

10 **a** **i** $\frac{300}{500} \times \frac{180}{500} \times 500 = 108$

ii $b = 12$ $c = 132$ $d = 88$

b H_0: position in upper management is independent of gender

H_1: position in upper management is dependent on gender

c **i** $\chi^2_{calc} = 54.9$

ii $(2 - 1)(3 - 1) = 2$

iii $54.9 > 5.991$, therefore we reject the null hypothesis. There is enough evidence to conclude that position in upper management is dependent on gender.

11 **a** H_1: The choice of candidate is dependent on where the voter lives.

b $\frac{3720}{8000} \times \frac{3680}{8000} \times 8000 = 1711$

c **i** $\chi^2_{calc} = 58.4$

ii $(3 - 1)(2 - 1) = 2$

d **i** The choice of candidate is dependent on gender.

ii $58.4 > 9.21$, therefore we reject the null hypothesis.

12 **a** $\frac{90}{200} \times \frac{110}{200} \times 200 = 49.5$

b **i** H_0: Grade is independent of gender

ii $(2 - 1)(3 - 1) = 2$

iii $\chi^2_{calc} = 0.400$

c $0.400 < 5.991$, therefore we do not reject the null hypothesis. There is enough evidence to conclude that grade is independent of gender

6 Introducing differential calculus

Answers

Skills check

1 a $f(5) = -7$, $f(-5) = 13$
b $f(2) = 11$, $f(-3) = -4$
c $g(5) = 25$, $g\left(\frac{1}{2}\right) = \frac{1}{4}$
d $g(2) = 1\frac{1}{2}$, $g(15) = \frac{1}{5}$
e $f(4) = 3.2$, $f(-3) = -4.5$

2 a $r = \frac{c}{2\pi}$ b $r = \pm\sqrt{\frac{A}{\pi}}$ c $r = \pm\sqrt{\frac{A}{4\pi}}$
d $r = \pm\sqrt{\frac{3V}{\pi h}}$ e $r = \pm\sqrt[3]{\frac{3V}{2\pi}}$ f $r = \frac{2A}{C}$

3 a $4^2 = 16$ b $2^{-3} = \frac{1}{2^3} = \frac{1}{8}$
c $\left(\frac{1}{2}\right)^4 = \frac{1^4}{2^4} = \frac{1}{16}$

4 a x^{-1} b x^{-4} c x^2 d x^{-3} e x

5 a $y + 3 = 2(x - 5) \Rightarrow y = 2x - 13$
b $y - 2 = -3(x - 4) \Rightarrow y = -3x + 14$

Exercise 6A

1 a $8x$ b $18x^2$ c $28x^3$ d $15x^2$
e $4x^3$ f 5 g 1 h 12
i $18x$ j $\frac{3}{2}x^2$ k x e $3x^3$

2 a 0 b $-9x^2$ c $-x^3$ d $-2x^2$
e -1 f 0 g $30x^5$ h $-63x^8$
i $4x^7$ j $9x^{11}$ k $-6x^8$ e 0

3 a $6x + 15x^2$ b $20x^3 - 4$
c $9 - 33x^2$ d $4x^3 + 3$

4 a $-5 + 24x^5$ b $18x - 5$
c $7 + 20x^4$ d $4x + 3$

Exercise 6B

1 a $A = 36t - 4t^3 \Rightarrow \frac{dA}{dt} = 36 - 12t^2$
b $A = 12t + 30 \Rightarrow \frac{dA}{dt} = 12$
c $A = t^3 - 5t^2 \Rightarrow \frac{dA}{dt} = 3t^2 - 10t$
d $A = 2t^2 + t - 6 \Rightarrow \frac{dA}{dt} = 4t + 1$
e $A = 15 + 7t - 2t^2 \Rightarrow \frac{dA}{dt} = 7 - 4t$
f $A = 18t^2 - 9t - 35 \Rightarrow \frac{dA}{dt} = 36t - 9$
g $A = t^3 - t^2 + 3t - 3 \Rightarrow \frac{dA}{dt} = 3t^2 - 2t + 3$
h $A = 3t^2 - 3t - 36 \Rightarrow \frac{dA}{dt} = 6t - 3$

2 a $f(r) = \frac{1}{2}(2r^2 - 18) = f'(r) = 2r$
b $f'(r) = 2(r + 3) = 2r + 6$
c $f'(r) = 2(2r - 3) \times 2 = 4(2r - 3) = 8r - 12$
d $f'(r) = 2(5 - 2r) \times -2 = -4(5 - 2r) = 8r - 20$
e $f'(r) = 6(r + 5) = 6r + 30$
f $f'(r) = 5 \times 2(7 - r) \times -1 = -10(7 - r) = 10r - 70$

Exercise 6C

1 $\frac{dy}{dx} = -\frac{6}{x^3}$

2 $f'(x) = -\frac{8}{x^5}$

3 $\frac{dy}{dx} = -\frac{7}{x^2}$

4 $f'(x) = -\frac{16}{x^9}$

5 $\frac{dy}{dx} = -\frac{35}{x^8}$

6 $\frac{dy}{dx} = -\frac{2}{x^2}$

7 $f'(x) = 14x - \frac{20}{x^6}$

8 $\frac{dy}{dx} = -4 - \frac{5}{x^3}$

9 $g'(x) = 3x^2 - \frac{6}{x^3}$

10 $\frac{dy}{dx} = 4 + \frac{3}{x^2}$

11 $g'(x) = 15x^2 + \frac{4}{x^5}$

12 $\frac{dy}{dx} = 2x^3 + \frac{6}{x^9}$

13 $\frac{dy}{dx} = \frac{x^3}{2} + 6x - \frac{10}{3x^5}$

14 $g'(x) = 6x^2 - 2x + \frac{3}{x^3}$

15 $A'(x) = 2x + \frac{5}{2x^2} - \frac{3}{2x^3}$

Exercise 6D

1 $\frac{dy}{dx} = 2x - 3$
when $x = 4$, $\frac{dy}{dx} = 2(4) - 3 = 5$

2 $\frac{dy}{dx} = 6 - 3x^2$
when $x = 0$, $\frac{dy}{dx} = 6 - 3(0)^2 = 6$

3 $\frac{dy}{dx} = -8x^3 - 9x^2$

when $x = -3$, $\frac{dy}{dx} = -8(-3)^3 - 9(-3)^2 = 135$

4 $y = 10x^2 + 8x \Rightarrow \frac{dy}{dx} = 20x + 8$

when $x = -1$, $\frac{dy}{dx} = 20(-1) + 8 = -12$

5 $\frac{dy}{dx} = 3x^2 - 5$

when $x = 6$, $\frac{dy}{dx} = 3(6)^2 - 5 = 103$

6 $\frac{dy}{dx} = -2x^3$

when $x = -2$, $\frac{dy}{dx} = -2(-2)^3 = 16$

7 $\frac{dy}{dx} = 21 - 36x^2$

when $x = 1$, $\frac{dy}{dx} = 21 - 36(1)^2 = -15$

8 $\frac{dy}{dx} = 6x - 5$

At $(-2, 28)$, $\frac{dy}{dx} = 6(-2) - 5 = -17$

9 $\frac{ds}{dt} = 40 - 10t$

At $t = 0$, $\frac{ds}{dt} = 40 - 10(0) = 40$

10 $s = 35t + 6t^2 \Rightarrow \frac{ds}{dt} = 35 + 12t = 35$ at $t = 0$

At $t = 3$, $\frac{ds}{dt} = 35 + 12(3) = 71$

11 $\frac{dv}{dt} = 80$

12 $\frac{dv}{dt} = 0.7$

13 $\frac{dA}{dh} = 42h^2$. At $h = \frac{2}{3}$, $\frac{dA}{dh} = 42 \times \frac{4}{9} = \frac{14 \times 4}{3} = \frac{56}{3}$

14 $\frac{dW}{dp} = 21.75p^2$. When $p = -2$, $\frac{dW}{dp} = 21.75 \times 4 = 87$

15 $\frac{dV}{dr} = 8r - \frac{18}{r^2}$. When $r = 3$, $\frac{dV}{dr} = 24 - \frac{18}{9} = 22$

16 $\frac{dA}{dr} = 5 - \frac{16}{r^3}$. When $r = 4$, $\frac{dA}{dr} = 5 - \frac{16}{64} = 4\frac{3}{4}$

17 $\frac{dV}{dr} = 21r^2 + \frac{8}{r^2}$. When $r = 2$, $\frac{dV}{dr} = 21 \times 4 + \frac{8}{4} = 86$

18 $\frac{dA}{dr} = 2\pi r + \frac{2\pi}{r^2}$. When $r = 1$, $\frac{dV}{dr} = 4\pi$ at $r = 1$

19 $\frac{dV}{dr} = 6 - \frac{15}{2r^2}$. When $r = 5$, $\frac{dV}{dr} = 6 - \frac{\cancel{15}^{3}}{\cancel{50}_{10}} = 5\frac{2}{10}$

20 $\frac{dC}{dr} = 45 - \frac{36}{r^4}$. When $r = 1$, $\frac{dC}{dr} = 45 - 36 = 9$

Exercise 6E

1 **a** $\frac{dy}{dx} = 2x + 3$

b At P, $\frac{dy}{dx} = 2x + 3 = 7$

$2x = 4$

$x = 2$

c At P, $y = (2)^2 + 3(2) - 4$

$= 4 + 6 - 4$

$= 6$

2 **a** $\frac{dy}{dx} = 4x + 1$

b At Q, $\frac{dy}{dx} = 4x + 1 = -9$

$4x = -10$

$x = -\frac{10}{4} = -\frac{5}{2}$

c At Q, $y = 2\left(\frac{-5}{2}\right)^2 - \left(\frac{-5}{2}\right) + 1$

$= \frac{25}{2} + \frac{5}{2} + 1$

$= 16$

3 **a** $\frac{dy}{dx} = 3 - 2x$

b At R, $\frac{dy}{dx} = 3 - 2x = -3$

$6 = 2x$

$x = 3$

Also, $y = 4 + 3(3) - (3)^2$

$= 4 + 9 - 9$

$= 4$

So, R = (3, 4)

4 $\frac{dy}{dx} = 2x - 6$

At R, $\frac{dy}{dx} = 2(a) - 6 = 6$

$2a = 12$

$a = 6$

Also, $y = (6)^2 - 6(6) = 0$

So R is (6, 0)

5 $\frac{dy}{dx} = 6x + 1 = 4$ when gradient is 4.

$\Rightarrow 6x = 3 \quad \Rightarrow x = \frac{1}{2}, y = -3\frac{3}{4}$

point is $\left(\frac{1}{2}, -3\frac{3}{4}\right)$

6 $\frac{dy}{dx} = 5 - 4x$

when gradient is 9

$\frac{dy}{dx} = 5 - 4x = 9$

$-4 = 4x$

$x = -1$

Also, $y = 5(-1) - 2(-1)^2 - 3$

$= -5 - 2 - 3$

$= -10$

So the point is (–1, –10)

7 $\frac{dy}{dx} = 3x^2 + 3$

when gradient is 6, $\frac{dy}{dx} = 3 + 3 = 6$

$3x^2 = 3$

$x^2 = 1$

$x = \pm 1$

when $x = 1$

$y = (1)^3 + 3(1) + 4 = 8$

when $x = -1$

$y = (-1)^3 + 3(-1) + 4 = 0$

So points are (1, 8) and (–1, 0)

8 $\frac{dy}{dx} = 3x^2 - 6$

when gradient is –3, $\frac{dy}{dx} = -3$

$\Rightarrow 3x^2 - 6 = -3$

$x^2 = 1$

$x = \pm 1$

At $x = 1$, $y = (1)^3 - 6(1) + 1 = -4$

At $x = -1$, $y = (-1)^3 - 6(-1) + 1 = 6$

So points are (1, –4) and (–1, 6)

straight line has gradient $m = \frac{6-(-4)}{-1-1}$

$m = \frac{10}{-2} = -5$

line has eqn $y - (-4) = -5(x - 1)$

$\Rightarrow y + 4 = -5x + 5$

$y = -5x + 1$

9 $\frac{dy}{dx} = 3x^2 - 12$

$0 = 3x^2 - 12$ when gradient is zero

$x^2 = 4$

$x = \pm 2$

At $x = 2$, $y = (2)^3 - 12(2) + 5 = -11$

At $x = -2$, $y = (-2)^3 - 12(-2) + 5 = 21$

The 2 points are (2, –11) and (–2, 21)

straight line has gradient $m = \frac{21-(-11)}{-2-2}$

$m = \frac{32}{-4} = -8$

straight line is $y - (-11) = -8(x - 2)$

$y + 11 = -8x + 16$

$y = -8x + 5$

10 a $b = (1)^2 - 4(1) + 1 = -2$

b $2x - 4$

c At P, $\frac{dy}{dx} = 2 - 4 = -2 = b$

d At Q, $\frac{dy}{dx} = 2x - 4 = -2$

$\Rightarrow x = 1 \Rightarrow y = 1 - 4 + 1 = -2$ so $d = -2$

11 a $b = 25 - 15 - 3 = 7$

b $\frac{dy}{dx} = 2x - 3$

c At P, $\frac{dy}{dx} = 2 \times 5 - 3 = 7 = b$

d At Q, $\frac{dy}{dx} = 2x - 3 = -3$

$\Rightarrow x = 0 \Rightarrow y = -3 \Rightarrow d = -3$

12 a $f'(x) = 4 - 2x$

b At $x = 5$, $f'(x) = 4 - 10 = 6$

or $f(x) = 20 - 25 - 1 = -6$

so $f(x) = f'(x)$

c $f(x) = f'(x) \Rightarrow 4x - x^2 - 1 = 4 - 2x$

$\Rightarrow 0 = x^2 - 6x + 5$

$= (x - 5)(x - 1)$

Second point is (1, 2)

13 a $f'(x) = 4x - 1$

b At $x = 2$, $f'(x) = 8 - 1 = 7$ and

$f(x) = 8 - 2 + 1 = 7$ so $f(x) = f'(x)$

c $f(x) = f'(x) \Rightarrow 2x^2 - x + 1 = 4x - 1$

$\Rightarrow 2x^2 - 5x + 2 = 0$

$\Rightarrow (2x - 1)(x - 2) = 0$

Second point is $(\frac{1}{2}, 1)$

14 a $f'(x) = 3 - 2x$

b At $x = 1$, $f'(x) = 3 - 2 = 1$ and

$f(x) = 3 - 1 - 1$ so $f(x) = f'(x)$

c $f(x) = f'(x) \Rightarrow 3 - 2x = 3x - x^2 - 1$

$\Rightarrow x^2 - 5x + 4 = 0$

$\Rightarrow (x - 4)(x - 1) = 0$

$\Rightarrow$ Second point is (4, –5)

15 a $f'(x) = 4x - 1$

b $f(x) = f'(x) \Rightarrow 2x^2 - x - 1 = 4x - 1$

$\Rightarrow 2x^2 - 5x = 0$

$\Rightarrow x(2x - 5) = 0$

$\Rightarrow x = 0$ or $\frac{5}{2}$

$\Rightarrow (0, -1)$ and $(\frac{5}{2}, 9)$

16 a $f'(x) = 2x + 5$

b $x^2 + 5x - 5 = 2x + 5$

$\Rightarrow x^2 + 3x - 10 = 0$

$\Rightarrow (x + 5)(x - 2) = 0$

points are (–5, –5) and (2, 9)

17 $x^2 + 4x + 5 = 2x + 4$

$\Rightarrow x^2 + 2x + 1 = 0$

$\Rightarrow (x + 1)^2 = 0$

$\Rightarrow x = -1$

$\Rightarrow (-1, 2)$

Exercise 6F

1 a $\frac{dy}{dx} = 2x = 6$ at $x = 3$

tangent is $y - 9 = 6(x - 3) \Rightarrow y = 6x - 9$

b $\frac{dy}{dx} = 6x^2 = 6$ at $x = 1$

tangent is $y - 2 = 6(x - 1) \Rightarrow y = 6x - 4$

c $\frac{dy}{dx} = 6 - 2x = 2$ at $x = 2$

tangent is $y - 8 = 2(x - 2) \Rightarrow y = 2x + 4$

d $\frac{dy}{dx} = 6x = 6$ at $x = 1$

tangent is $y + 7 = 6(x - 1) \Rightarrow y = 6x - 13$

e $\frac{dy}{dx} = 4x - 5 = 7$ at $x = 3$

tangent is $y - 7 = 7(x - 3) \Rightarrow y = 7x - 14$

f $\frac{dy}{dx} = 10 - 3x^2 = -2$ at $x = 2$

tangent is $y - 17 = -2(x - 2) \Rightarrow y = -2x + 21$

g $\frac{dy}{dx} = -4x = -12$ at $x = 3$
tangent is $y + 7 = -12(x - 3) \Rightarrow y = -12x + 29$

h $\frac{dy}{dx} = -2x + 6 = 2$ at $x = 2$
tangent is $y - 13 = 2(x - 2) \Rightarrow y = 2x + 9$

i $\frac{dy}{dx} = 8x - 3x^2 = 32 - 48 = -16$ at $x = 4$
tangent is $y - 0 = -16(x - 4) \Rightarrow y = -16x + 64$

j $\frac{dy}{dx} = 5 - 6x = 11$ at $x = -1$
tangent is $y + 8 = 11(x + 1) \Rightarrow y = 11x + 3$

k $\frac{dy}{dx} = 12x - 6x^2 = 24 - 24 = 0$ at $x = 2$
tangent is $y = 8$

l $\frac{dy}{dx} = 60 - 10x = 40$ at $x = 2$
tangent is $y - 107 = 40(x - 2)\ y = 40x + 27$

m $\frac{dy}{dx} = 2x^3 = 128$ at $x = 4$
tangent is $y - 121 = 128(x - 4) \Rightarrow y = 128x - 391$

n $\frac{dy}{dx} = -3 + 10x = -3$ at $x = 0$
tangent is $y - 17 = -3x \Rightarrow y = -3x + 17$

o $\frac{dy}{dx} = 10 - 4x = 10$ at $x = 0$
tangent is $y - 0 = 10\,(x - 0) \Rightarrow y = 10x$

p $\frac{dy}{dx} = \frac{3x^2}{4} - 4 = -1$ at $x = 2$
tangent is $y + 6 = -1\,(x - 2) \Rightarrow y = -x - 4$

q $\frac{dy}{dx} = \frac{3x}{2} - 3$ at $x = -2$
tangent is $y - 6 = -3\,(x + 2) \Rightarrow y = -3x$

r $\frac{dy}{dx} = 2x^2 = 2$ at $x = -1$
tangent is $y + \frac{1}{3} = 2\,(x + 1) \Rightarrow y = 2x + 1\frac{2}{3}$

s $\frac{dy}{dx} = \frac{3}{4}x^2 - 14x = 31$ at $x = -2$
tangent is $y + 25 = 31\,(x + 2) \Rightarrow y = 31x + 37$

2 a $\frac{dy}{dx} = -\frac{24}{x^3} = -3$ at $x = 2$
tangent is $y - 3 = -3\,(x - 2) \Rightarrow y = -3x + 9$
$\Rightarrow 3x + y - 9 = 0$

b $\frac{dy}{dx} = -\frac{18}{x^4} = -18$ at $x = 1$
tangent is $y - 11 = -18\,(x - 1) \Rightarrow y = -18x + 29$
$\Rightarrow 18x + y - 29 = 0$

c $\frac{dy}{dx} = 6 + \frac{16}{x^3} = 4$ at $x = -2$
tangent is $y + 14 = 4\,(x + 2) \Rightarrow 4x - y - 6 = 0$

d $\frac{dy}{dx} = 3x^2 - \frac{12}{x^3} = 15$ at $x = -1$
tangent is $y - 5 = 15\,(x + 1) \Rightarrow 15x - y + 20 = 0$

e $\frac{dy}{dx} = 5 + \frac{8}{x^2} = 5\frac{1}{2} = \frac{11}{2}$ at $x = 4$
tangent is $y - 18 = \frac{11}{2}\,(x - 4)$
$\Rightarrow 2y - 36 = 11x - 44 \Rightarrow 11x - 2y - 8 = 0$

Exercise 6G

1 $\frac{dy}{dx} = 4x = 4$ at $x = 1 \Rightarrow m' = -\frac{1}{4}$
Normal is $y - 2 = -\frac{1}{4}\,(x - 1)$
$\Rightarrow 4y - 8 = -x + 1$
$\Rightarrow x + 4y - 9 = 0$

2 $\frac{dy}{dx} = 12x^2 = 3$ at $x = \frac{1}{2} \Rightarrow m' = -\frac{1}{3}$
Normal is $y - \frac{7}{2} = -\frac{1}{3}\left(x - \frac{1}{2}\right)$
$\Rightarrow 3y - \frac{21}{2} = -x + \frac{1}{2}$
$\Rightarrow x + 3y - 11 = 0$

3 $\frac{dy}{dx} = \frac{1}{2} - 2x = -3\frac{1}{2} = -\frac{7}{2}$ at $x = 2 \Rightarrow m' = \frac{2}{7}$
Normal is $y + 3 = \frac{2}{7}\,(x - 2)$
$\Rightarrow 7y + 21 = 2x - 4$
$\Rightarrow 2x - 7y - 25 = 0$

4 $\frac{dy}{dx} = 3x + 1 = 5$ at $x = -2 \Rightarrow m' = \frac{1}{5}$
Normal is $y - 4 = \frac{1}{5}(x - 2)$
$\Rightarrow 5y - 20 = x + 2$
$\Rightarrow x - 5y + 22 = 0$

5 $y = 10 + 3x - x^2$
$\frac{dy}{dx} = 3 - 2x = 3$ at $x = 0 \Rightarrow m' = -\frac{1}{3}$
Normal is $y - 10 = -\frac{1}{3}x \Rightarrow x + 3y - 30 = 0$

6 $\frac{dy}{dx} = 2(x + 2) = 4$ at $x = 0 \Rightarrow m' = -\frac{1}{4}$
Normal is $y - 4 = -\frac{1}{4}x \Rightarrow x + 4y - 16 = 0$

7 $\frac{dy}{dx} = -\frac{4}{x^2} = -1$ at $x = 2 \Rightarrow m' = 1$
Normal is $y - 2 = x - 2$ i.e., $x - y = 0$

8 $\frac{dy}{dx} = -\frac{12}{x^3} = 12$ at $x = -1 \Rightarrow m' = -\frac{1}{12}$
Normal is $y - 6 = \frac{-1}{12}(x + 1) \Rightarrow 12y - 72 = -x - 1$
$\Rightarrow x + 12y - 71 = 0$

9 $\frac{dy}{dx} = 6 - \frac{8}{x^2} = -2$ at $x = -1 \Rightarrow m' = -\frac{1}{12}$
Normal is $y - 14 = \frac{1}{2}(x - 1) \Rightarrow 2y - 28 = x - 1$
$\Rightarrow x - 2y + 27 = 0$

10 $\frac{dy}{dx} = 4x^3 + \frac{9}{x^4} = 5$ at $x = -1 \Rightarrow m' = -\frac{1}{5}$
Normal is $y - 4 = -\frac{1}{5}(x + 1) \Rightarrow 5y - 20 = -x - 1$
$\Rightarrow x + 5y - 19 = 0$

11 $\frac{dy}{dx} = -2 + \frac{1}{x^2} = 2$ at $x = \frac{1}{2} \Rightarrow m' = -\frac{1}{2}$
Normal is $y - 1 = -\frac{1}{2}\left(x - \frac{1}{2}\right) \Rightarrow 2y - 2 = -x + \frac{1}{2}$
$\Rightarrow 4y - 4 = -2x + 1$
$\Rightarrow 2x + 4y - 5 = 0$

12 $\frac{dy}{dx} = 5 + \frac{9}{2x^2} = 5\frac{1}{2}$ at $x = 3 \Rightarrow m' = -\frac{2}{11}$
Normal is $y - 13.5 = -\frac{2}{11}(x - 3)$
$\Rightarrow 11y - 148.5 = -2x + 6$
$\Rightarrow 22y - 297 = -4x + 12$
$\Rightarrow 4x + 22y - 309 = 0$

Exercise 6H

1 $\frac{dy}{dx} = 2(x-4) = 2$ at $x = 5$. At $x = 5$, $y = (5-4)^2 = 1$

At (5, 1), tangent is $y - 12(x - 5) \Rightarrow y = 2x - 9$

2 $y = x^3 - 3x \Rightarrow \frac{dy}{dx} = 3x^2 - 3 = 9$ at $x = -2$

At $x = -2$, $y = -8 + 6 = -2$

tangent is $y + 2 = 9(x + 2) \Rightarrow y = 9x + 16$

3 $\frac{dy}{dx} = 1 - \frac{6}{x^2} = 1 - \frac{3}{8} = \frac{5}{8}$ at $x = 4 \Rightarrow m' = -\frac{8}{5}$ at $x = 4$

$y = 4 + \frac{6}{4} = 5\frac{1}{2}$ at $x = 4$

Normal is $y - 5\frac{1}{2} = -\frac{8}{5}(x-4)$

$\Rightarrow 10y + 16x - 119 = 0$

4 $\frac{dy}{dx} = 2x + \frac{2}{x^3} = -4$ at $x = -1 \Rightarrow m' = \frac{1}{4}$

$x = -1 \Rightarrow y = 1 - 1 = 0$

Normal is $y = \frac{1}{4}(x+1) \Rightarrow 4y - x - 1 = 0$

5 $y = 8 \Rightarrow 3x^2 - 2x - 8 = 0$

$\Rightarrow (3x + 4)(x - 2) = 0$

$\Rightarrow x = -\frac{4}{3}$ or $x = 2$

$$\frac{dy}{dx} = 6x - 2 = \begin{cases} -10 & \text{at } x = -\frac{4}{3} \\ 10 & \text{at } x = 2 \end{cases}$$

tangents are $y - 8 = -10\left(x + \frac{4}{3}\right)$

$\Rightarrow 3y - 24 = -30x - 40$

$\Rightarrow 3y + 30x + 16 = 0$

and $y - 8 = 10(x - 2)$

$\Rightarrow y = 10x - 12$

6 $y = 6x - 2x^2 = -20 \Rightarrow 3x - x^2 = -10$

$\Rightarrow x^2 - 3x - 10 = 0$

$\Rightarrow (x - 5)(x + 2) = 0$

$\Rightarrow x = 5$ or -2

$$\frac{dy}{dx} = 6 - 4x = \begin{cases} -14 & \text{at } x = 5 \\ 14 & \text{at } x = -2 \end{cases}$$

tangents are $y + 20 = -14(x - 5) \Rightarrow y = -14x + 50$

and $y + 20 = 14(x + 2) \Rightarrow y = 14x + 8$

7 $y = 7 - 5x - 2x^3$

when $y = 0$, $7 - 5x - 2x^3 = 0$

Try $x = 1$: $7 - 5(1) - 2(1) = 7 - 5 - 2 = 0$

so the curve intersects the x-axis at $x = 1$

$\frac{dy}{dx} = -5 - 6x^2$

At $x = 1$, $\frac{dy}{dx} = -5 - 6 = -11$

so normal at (1, 0) has gradient $m' = \frac{1}{11}$

Thus, $y - 0 = \frac{1}{11}(x - 1)$

$11y = x - 1$

$11y - x + 1 = 0$

8 $y = x^3 + 3x - 2$

At $y = -6$, $x^3 + 3x - 2 = -6$

$x^3 + 3x + 4 = 0$

Try $x = -1$: $(-1)^3 + 3(-1) + 4 = -1 - 3 + 4 = 0$

so the curve passes through $(-1, -6)$

$\frac{dy}{dx} = 3x^2 + 3$

At $x = -1$, $\frac{dy}{dx} = 3 + 3 = 6$

so normal at $(-1, -6)$ has gradient $m' = -\frac{1}{6}$

Thus, $y - (-6) = -\frac{1}{6}(x - (-1))$

$y + 6 = -\frac{1}{6}(x + 1)$

$6y + 36 = -x - 1$

$6y + x + 37 = 0$

9 **a** $\frac{dy}{dx} = 0 \Rightarrow 2(4x - 3) \times 4 = 0 \Rightarrow x = \frac{3}{4}$

b At $x = \frac{3}{4}$, $y = 0$. So

Tangent is $y = 0(x) + c$

$0 = 0\left(\frac{3}{4}\right) + c \Rightarrow c = 0$, so $y = 0$ is the tangent

10 **a** $y = x^2 + 16x^{-1}$

$\frac{dy}{dx} = 2x - 16x^{-2}$

$\frac{dy}{dx} = 2x - \frac{16}{x^2}$

$0 = 2x - \frac{16}{x^2}$

$2x = \frac{16}{x^2}$

$2x^3 = 16$

$x^3 = 8$

$x = 2$

b $y = 2^2 + \frac{16}{2} = 12$

Since the gradient is zero, the equation of the tangent is $y = 12$

11 **a** $\frac{dy}{dx} = x + 1 = 5 \Rightarrow x = 4$

At $x = 4$, $y = 8 + 4 - 3 = 9$. So tangent is:

$y - 9 = 5(x - 4) \Rightarrow y = 5x - 11$

12 **a** $\frac{dy}{dx} = 4x^3 + 3 = 3 \Rightarrow x = 0$

b At $x = 0$, $y = -3$. So tangent is $y + 3 = 3x$

$\Rightarrow y = 3x - 3$

c Normal is $y + 3 = -\frac{1}{3}(x - 0) \Rightarrow 3y + x + 9 = 0$

13 **a** $\frac{dy}{dx} = 4 - \frac{12}{x^5} = 16 \Rightarrow x^5 = -1 \Rightarrow x = -1$

b At $x = -1$, $y = -4 + 3 = -1$. Tangent is

$y + 1 = 16(x + 1) \Rightarrow y = 16x + 15$

c Normal is $y + 1 = -\frac{1}{16}(x + 1)$

$\Rightarrow 16y + 2x + 17 = 0$

14 $\frac{dy}{dx} = 6x^2 + 18x - 24 = 36$

$\Rightarrow 3x^2 + 9x - 12 = 18$

$\Rightarrow 3x^2 + 9x - 30 = 0$

$\Rightarrow x^2 + 3x - 10 = 0$

$\Rightarrow x = -5 \quad \text{or} \quad 2$

$y = 100 \quad \text{or} \quad 9$

Tangents are $y - 100 = 36(x + 5) \Rightarrow y = 36x + 280$

and $y - 9 = 36(x - 2) \Rightarrow y = 36x - 63$

15 $\frac{dy}{dx} = 2x + k$

$\Rightarrow 6 + k = 7 \Rightarrow k = 1$

$\therefore y = x^2 + x$, so $b = 9 + 3 = 12$

16 $\frac{dy}{dx} = 2x + k = 1$ when $x = -2 \Rightarrow -4 + k = 1 \Rightarrow k = 6$

$\Rightarrow b = 4 - 2k = 4 - 10 = -6$

17 $\frac{dy}{dx} = 2kx - 2 = 2$ when $x = 4 \Rightarrow 8k = 4 \Rightarrow k = \frac{1}{2}$

$\Rightarrow b = 16k - 8 + 3 = 8 - 8 + 3 = 3$

18 $\frac{dy}{dx} = k - 3x^2 = -5$ when $x = -2$

$\Rightarrow k - 12 = -5 \Rightarrow k = 7$

$\Rightarrow b = 4 - 2k + 8 = 4 - 14 + 8 = -2$

19 $y = px^2 + qx \Rightarrow 4p + 2q = 5 \qquad (1)$

$\frac{dy}{dx} = 2px + q = 7$ at $x = 2 \Rightarrow 4p + q = 7 \qquad (2)$

$(1) - (2) \Rightarrow q = -2$ and $\therefore p = 2\frac{1}{4}$

20 $y = px^2 + qx - 5 \Rightarrow 9p - 3q - 5 = 13 \Rightarrow 9p - 3q = 18$

$\Rightarrow 3p - q = 6 \qquad (1)$

Also $\frac{dy}{dx} = 2px + q = 6$ at $x = -3$

$\Rightarrow -6p + q = 6 \qquad (2)$

$(1) + (2) \Rightarrow -3p = 12 \Rightarrow p = -4$ and $q = -18$

Exercise 6I

1 **a** $V(0) = 100\ \text{cm}^3$

b $V(3) = 100 + 6 + 27 = 133\ \text{cm}^3$

c $\frac{dV}{dt}$ represents the rate of change of the volume of water in the container.

d $\frac{dV}{dt} = 2 + 3t^2 = 2 + 27 = 29\ \text{cm}^3/\text{sec}$ when $t = 3$

e There is 133 cm³ of water in the container when $t = 3$ and the container and, at that time, water is flowing into the container at $29\ \text{cm}^3 s^{-1}$.

2 **a** $A(0) = 0\ \text{cm}^2$

b $A(5) = 45\ \text{cm}^2$

c $\frac{dA}{dt}$ represents the rate of change of the area of the pool

d $A = 4t + t^2 \Rightarrow \frac{dA}{dt} = 4 + 2t = 14\ \text{cm}^2/\text{sec}$ when $t = 5$

e The pool has reached an area of 45 cm² when $t = 5$ and, at this time, the area is increasing at $14\ \text{cm}^2\text{s}^{-1}$

3 **a** $W(1) = 5 + 640 + 40 = 685$ tonnes

b $\frac{dW}{dt} = 10t - \frac{640}{t^2}$

c **i** $\frac{dW}{dt}(3) - 41\frac{1}{9}$ tonnes/hr.

ii $\frac{dW}{dt}(5) = 24\frac{2}{5}$ tonnes/hr.

d The tank was emptying when $t = 3$, but has now started filling again at $t = 5$

e $10t^3 = 640 \Rightarrow t = 4$ hours.

f This is the time at which the weight of the oil in the tank reaches its minimum value.

4 **a** $\frac{dV}{dt} = 6 + 2t = 8\ \text{m}^3/\text{min}$, when $t = 1$.

b $V = 65 \Rightarrow t^2 + 6t + 10 = 65 \Rightarrow t^2 + 6t - 55 = 0$

$\Rightarrow (t + 11)(t - 5) = 0$

$\Rightarrow t = 5$ (must be positive)

$\Rightarrow \frac{dV}{dt} = 6 + 2t = 16\ \text{m}^3/\text{min}$.

5 **a** $\frac{dy}{dt} = -4 - 3t^2 = \begin{cases} -16\ \text{cm/sec.} & \text{when } t = 2 \\ -31\ \text{cm/sec.} & \text{when } t = 3 \end{cases}$

At $t = 2$, depth is decreasing at 16 cm/sec.

At $t = 3$, depth is decreasing at 31 cm/sec.

b $y = 0$ when $t^3 + 4t - 500 = 0$

$\Rightarrow t = 7.8$ secs (1 d.p.)

6 **a** $\frac{dA}{dt} = \frac{3t}{2} + \frac{1}{2} = 3\frac{1}{2}\ \text{cm}^2/\text{sec}$ when $t = 2$

b $A = 30 \Rightarrow \frac{3t^2}{4} + \frac{t}{2} = 30$

$\Rightarrow 3t^2 + 2t - 120 = 0$

$\Rightarrow (3t + 20)(t - 6) = 0$

$\Rightarrow t = 6$ (must be positive)

$\Rightarrow \frac{dA}{dt}(6) = \frac{3\times 6}{2} + \frac{1}{2} = 9\frac{1}{2}\ \text{cm}^2/\text{sec}$.

when $t = 6$

7 **a** $\frac{dW}{dt} = 10 - \frac{270}{t^3} = 10 - \frac{270}{8}$

$= -23.75$ tonnes/hour when $t = 2$

b $\frac{dW}{dt} = 0 \Rightarrow t^3 = 27 \Rightarrow t = 3$ hours

8 **a** $\frac{d\theta}{dt} = 12t^2 - 2t = 12(2)^2 - 2(2) = 44$ degrees/sec when $t = 2$.

b $\frac{d\theta}{dt} = 0 \Rightarrow 2t(6t - 1) = 0 \Rightarrow t = \frac{1}{6}$ sec.

9 **a** $P(0) = -15$ i.e. there is a 15 000 dollar start-up cost

$P(5) = -215$. The company makes a loss of 215 000 dollars if it produces 5 tonnes of product

b $\frac{dP}{dx} = -30x^2 + 80x + 10$ dollars/tonne

c **i** When $x = 2$, $P = 85$ and $\frac{dP}{dx} = 50$

ii When $x = 3$, $P = 105$ and $\frac{dP}{dx} = -20$

d The company is in profit when 2 tonnes are made and as production increases, profit increases, but although a greater profit is made when 3 tonnes are produced, increasing production further will cause profit to fall.

e $\frac{dP}{dx} = 0 \Rightarrow 30x^2 - 80x - 10 = 0$

$\Rightarrow 3x^2 - 8x - 1 = 0$

$\Rightarrow x = \frac{8 \pm \sqrt{64+12}}{6}$

x must be positive, so $x = 2.79$ tonnes (3 sf) giving a maximum profit of 107 088 dollars when 2.79 tonnes are made.

Exercise 6J

1 $\frac{dy}{dx} = 2x - 6 = 0$ when $x = 3$

2 $\frac{dy}{dx} = 12 - 4x = 0$ when $x = 3$

3 $\frac{dy}{dx} = 2x + 10 = 0$ when $x = -5$

4 $\frac{dy}{dx} = 6x + 15 = 0 \Rightarrow x = -\frac{5}{2}$

5 $\frac{dy}{dx} = 3x^2 - 27 = 0 \Rightarrow x = \pm 3$

6 $\frac{dy}{dx} = 24 - 6x^2 = 0 \Rightarrow x^2 = 4 \Rightarrow x = \pm 2$

7 $\frac{dy}{dx} = 12x^2 - 3 \Rightarrow x^2 = \frac{1}{4} \Rightarrow x = \pm\frac{1}{2}$

8 $\frac{dy}{dx} = 3 - 48x^2 = 0 \Rightarrow x^2 = \frac{1}{16} \Rightarrow x = \pm\frac{1}{4}$

9 $\frac{dy}{dx} = 6x^2 - 18x + 12 = 0 \Rightarrow x^2 - 3x + 2 = 0$

$\Rightarrow (x-2)(x-1) = 0 \Rightarrow x = 1$ or 2

10 $\frac{dy}{dx} = 9 + 12x + 3x^2 = 0 \Rightarrow x^2 + 4x + 3 = 0$

$\Rightarrow (x+3)(x+1) = 0 \Rightarrow x = -3$ or -1

11 $\frac{dy}{dx} = 3x^2 - 6x - 45 = 0 \Rightarrow x^2 - 2x - 15 = 0$

$\Rightarrow (x-5)(x+3) = 0$

$\Rightarrow x = 5$ or -3

12 $\frac{dy}{dx} = 24x + 3x^2 + 36 = 0$

$\Rightarrow x^2 + 8x + 12 = 0$

$\Rightarrow (x+6)(x+2) = 0$

$\Rightarrow x = -6$ or -2

13 $\frac{dy}{dx} = 6x^2 - 12x = 0 \Rightarrow 6x(x-2) = 0$

$\Rightarrow x = 0$ or 2

14 $\frac{dy}{dx} = 60x - 15x^2 = 0 \Rightarrow x(4-x) = 0$

$\Rightarrow x = 0$ or 4

15 $\frac{dy}{dx} = 1 - \frac{1}{x^2} = 0 \Rightarrow x^2 = 1 \Rightarrow x = \pm 1$

16 $\frac{dy}{dx} = 1 - \frac{4}{x^2} = 0 \Rightarrow x^2 = 4 \Rightarrow x = \pm 2$

17 $\frac{dy}{dx} = 4 - \frac{9}{x^2} = 0 \Rightarrow x^2 = \frac{9}{4} \Rightarrow x = \pm\frac{3}{2}$

18 $\frac{dy}{dx} = 8 - \frac{1}{2x^2} = 0 \Rightarrow x^2 = \frac{1}{16} \Rightarrow x = \pm\frac{1}{4}$

19 $\frac{dy}{dx} = 27 - \frac{8}{x^3} \Rightarrow x^3 = \frac{8}{27} \Rightarrow x = \frac{2}{3}$

20 $\frac{dy}{dx} = 1 - \frac{1}{x^3} = 0 \Rightarrow x^3 = 1 \Rightarrow x = 1$

Exercise 6K

1 $\frac{dy}{dx} = 3x^2 - 18x + 24 = 0$

when $x^2 - 6x + 8 = 0$

$(x-4)(x-2) = 0$

Stationary points are (2, 0) and (4, –4)

$\frac{dy}{dx}(0) = 24 > 0;$

$\frac{dy}{dx}(3) = -3 < 0;$

$\frac{dy}{dx}(5) = 9 > 0$

So (2, 0) is a maximum

(4, –4) is a minimum

2 $\frac{dy}{dx} = 3x^2 + 12x + 9 = 0$

$\Rightarrow x^2 + 4x + 3 = 0$

$\Rightarrow (x+3)(x+1) = 0$

$\Rightarrow x = -3$ or $x = -1$

Stationary points are (–3, 5) (–1, 1)

$\frac{dy}{dx}(-4) > 0$; $\frac{dy}{dx}(-2) < 0$; $\frac{dy}{dx}(0) > 0$

So (–3, 5) is maximum

(–1, 1) is minimum

3 $\frac{dy}{dx} = 9 + 6x - 3x^2 = 0$

$\Rightarrow x^2 - 2x - 3 = 0$

$\Rightarrow (x-3)(x+1) = 0$

$\Rightarrow x = -1$ or $x = 3$

Stationary points are (–1, –5) and (3, 27)

$\frac{dy}{dx}(-2) < 0$; $\frac{dy}{dx}(0) > 0$; $\frac{dy}{dx}(4) < 0$

So (–1, –5) minimum

(3, 27) maximum

4 $\frac{dy}{dx} = 3x^2 - 6x = 0$

$\Rightarrow x(x-2) = 0$

Stationary points are (0, 5) and (2, 1)

$\frac{dy}{dx}(-1) > 0$; $\frac{dy}{dx}(1) < 0$; $\frac{dy}{dx}(3) > 0$

So (0, 5) maximum

(2, 1) minimum

5 $\frac{dy}{dx} = 27 - 3x^2 = 0$

$\Rightarrow x^2 = 9$

$\Rightarrow x = \pm 3$

Stationary points are (–3, –54) and (3, 54)

$\frac{dy}{dx}(-4) < 0$; $\frac{dy}{dx}(0) > 0$; $\frac{dy}{dx}(4) < 0$

So (–3, –54) minimum

(3, 54) maximum

6 $\frac{dy}{dx} = 18x - 3x^2 = 0$

when $3x(6 - x) = 0$

$\Rightarrow x = 0, 6$

Stationary points are (0, 0) (6, 108)

$\frac{dy}{dx}(-1) < 0$, $\frac{dy}{dx}(1) > 0$; $\frac{dy}{dx}(7) < 0$

So (0, 0) minimum

(6, 108) maximum

7 $\frac{dy}{dx} = 1 - \frac{1}{x^2} = 0 \Rightarrow x^2 = 1 \Rightarrow x = \pm 1$

Stationary points are (–1, –2) and (1, 2)

$\frac{dy}{dx}(-2) > 0$; $\frac{dy}{dx}\left(-\frac{1}{2}\right) < 0$; $\frac{dy}{dx}\left(\frac{1}{2}\right) < 0$; $\frac{dy}{dx}(2) > 0$

So (–1, 2) maximum

(1, 2) minimum

8 $\frac{dy}{dx} = 1 - \frac{9}{x^2} = 0 \Rightarrow x^2 = 9 \Rightarrow x = \pm 3$

Stationary points are (–3, –6) and (3, 6)

$\frac{dy}{dx}(-4) > 0$; $\frac{dy}{dx}(-2) < 0$; $\frac{dy}{dx}(2) < 0$; $\frac{dy}{dx}(4) > 0$

so (–3, –6) maximum

(3, 6) minimum

9 $\frac{dy}{dx} = \frac{1}{2} - \frac{8}{x^2} = 0 \Rightarrow x^2 = 16 \Rightarrow x = \pm 4$

$\Rightarrow$ Stationary points are (–4, –4) and (4, 4)

$\frac{dy}{dx}(-5) > 0$; $\frac{dy}{dx}(-3) < 0$; $\frac{dy}{dx}(3) < 0$; $\frac{dy}{dx}(5) > 0$

So (–4, –4) maximum

(4, 4) minimum

10 $\frac{dy}{dx} = \frac{-9}{x^2} + \frac{1}{4} = 0 \Rightarrow x^2 = 36 \Rightarrow x = \pm 6$

$\Rightarrow$ Stationary points are (–6, –3) and (6, 3)

$\frac{dy}{dx}(-7) > 0$; $\frac{dy}{dx}(-5) < 0$; $\frac{dy}{dx}(5) < 0$; $\frac{dy}{dx}(7) > 0$

So (–6, –3) maximum

(6, 3) minimum

11 $\frac{dy}{dx} = 2x + \frac{16}{x^2} = 0 \Rightarrow x^3 = -8 \Rightarrow x = -2$

Stationary point is (–2, 12)

$\frac{dy}{dx}(-3) < 0$; $\frac{dy}{dx}(-1) > 0$

so (–2, 12) minimum

12 $\frac{dy}{dx} = 9 - \frac{\overset{1}{\cancel{2}}}{\underset{3}{\cancel{6}}x^3} = 0 \Rightarrow 27x^3 = 1 \Rightarrow x = \frac{1}{3}$

Stationary point is $\left(\frac{1}{3}, 4\frac{1}{2}\right)$

$\frac{dy}{dx}\left(\frac{1}{4}\right) < 0$; $\frac{dy}{dx}(1) > 0$

so $\left(\frac{1}{3}, 4\frac{1}{2}\right)$ minimum

Exercise 6L

1 At turning points:

$\frac{dy}{dx} = 2x - 4 = 0$

$x = 2$

$y(2) = (2)^2 - 4(2) + 10$

$= 6$

$\frac{dy}{dx}(0) < 0$; $\frac{dy}{dx}(3) > 0 \Rightarrow (2, 6)$ is a minimum

2 At turning point:

$\frac{dy}{dx} = 18 - 6x = 0$

$x = 3$

$y(3) = 18(3) - 3(3)^2 + 2$

$= 29$

$\frac{dy}{dx}(0) > 0$; $\frac{dy}{dx}(4) < 0 \Rightarrow (3, 29)$ maximum

3 At turning point:

$\frac{dy}{dx} = 2x + 1 = 0$

$x = \frac{-1}{2}$

$y\left(\frac{-1}{2}\right) = \left(\frac{-1}{2}\right)^2 + \left(\frac{-1}{2}\right) - 3$

$= \frac{-13}{4}$

$\frac{dy}{dx}(-1) < 0$; $\frac{dy}{dx}(0) > 0 \Rightarrow \left(\frac{-1}{2}, \frac{-13}{4}\right)$ minimum

4 At turning points,

$\frac{dy}{dx} = -5 + 2x = 0$

$x = \frac{5}{2}$

$y\left(\frac{5}{2}\right) = 8 - 5\left(\frac{5}{2}\right) + \left(\frac{5}{2}\right)^2$

$= \frac{7}{4}$

$\frac{dy}{dx}(0) < 0$; $\frac{dy}{dx}(3) > 0 \Rightarrow \left(\frac{5}{2}, \frac{7}{4}\right)$ minimum

5 At turning points,

$\frac{dy}{dx} = 3 - 2x = 0$

$x = \frac{3}{2}$

$y\left(\frac{3}{2}\right) = 3\left(\frac{3}{2}\right) + 11 - \left(\frac{3}{2}\right)^2$

$= 13.25$

$\frac{dy}{dx}(0) > 0$; $\frac{dy}{dx}(2) < 0 \Rightarrow (1.5, 13.25)$ maximum

6 At turning points,

$\frac{dy}{dx} = -12x - 15 = 0$

$x = \frac{-5}{4}$

$y\left(\frac{-5}{4}\right) = 20 - 6\left(\frac{-5}{4}\right)^2 - 15\left(\frac{-5}{4}\right)$

$= \frac{235}{8}$

$\frac{dy}{dx}(-2) > 0; \frac{dy}{dx}(0) < 0 \Rightarrow \left(\frac{-5}{4}, \frac{235}{8}\right)$ maximum

7 $y = x^2 - 10x + 21$

$\frac{dy}{dx} = 2x - 10 = 0$ for turning points

$x = 5$

$y(5) = (5)^2 - 10(5) + 21$

$= -4$

$\frac{dy}{dx}(0) < 0; \frac{dy}{dx}(10) > 0 \Rightarrow (5, -4)$ minimum

8 $y = x^2 - 18x$

$\frac{dy}{dx} = 2x - 18 = 0$ for turning points

$x = 9$

$y(9) = (9)^2 - 18(9)$

$= -81$

$\frac{dy}{dx}(0) < 0; \frac{dy}{dx}(10) > 0 \Rightarrow (9, -81)$ minimum

9 $y = x^2 + 4x$

$\frac{dy}{dx} = 2x + 4 = 0$ for turning points

$x = -2$

$y(-2) = (-2)^2 + 4(-2)$

$= -4$

$\frac{dy}{dx}(-3) < 0; \frac{dy}{dx}(0) > 0 \Rightarrow (-2, -4)$ minimum

Exercise 6M

1 **a** $b = 7 + h$

b $A = (7 + h)h = 7h + h^2$

2 **a** $x = 10 - t$

b $V = 3(10 - t)t = 30t - 3t^2$

3 **a** $y = 5 - 2x$

b $P = x^2(5 - 2x) = 5x^2 - 10x^3$

4 **a** $R = \frac{1}{2}(25 + r)r^2$

b $R = \frac{1}{2}n(n - 25)^2$

5 $x + 5m = 100 \Rightarrow x = 100 - 5m$

a $L = 2m(m + 100 - 5m) = 2m\,(100 - 4m)$

b $L = 2m(m + x) = 2\left(\frac{100-x}{5}\right)\left(\frac{100-x}{5} + x\right)$

$= \frac{2}{25}(100 - x)(100 + 4x)$

6 **a** $V = \pi r^2 h = \pi r^2(17 - 2r)$

b $V = \pi r^2 h = \frac{\pi}{4}(17 - h)^2 h$

7 **a** $12x - 3 = 2c \Rightarrow c = \frac{12x - 3}{2}$

Hence $y = 5x^2 + \frac{12x-3}{2} = \frac{1}{2}(10x^2 + 12x - 3)$

b $\frac{dy}{dx} = \frac{1}{2}(20x + 12) = 10x + 6$

c Minimum value occurs at $x = -0.6$ and is

$y = 5 \times 0.36 + \frac{12 \times -0.6 - 3}{2}$

$= -3.3$

d $c = \frac{12x - 3}{2} = \frac{(12 \times -0.6) - 3}{2} = -5.1$

8 Given $N = 2n(5 - x)$ and $12n + 10x = 15$

$\Rightarrow 10x = 15 - 12n$

$\Rightarrow x = \frac{15 - 12n}{10}$

a $N = 2n\left(5 - \frac{(15 - 12n)}{10}\right) = \frac{2n}{10}(50 - 15 + 12n)$

$= \frac{n}{5}(35 + 12n) = \frac{1}{5}(35n + 12n^2)$

b $\frac{dN}{dn} = \frac{1}{5}(35 + 24n)$

c Occurs when $24n = -35 \Rightarrow n = \frac{-35}{24}$

Minimum value is $N = \frac{\overset{7}{\cancel{35}}}{\cancel{5} \times 24}\left(35 - \frac{35}{2}\right) = \frac{-7}{24} \times \frac{35}{2}$

$= -\frac{245}{48}$

d $x = \frac{15 - 12 \times \frac{-35}{24}}{10} = \frac{15 + \frac{35}{2}}{10} = 3.25$

9 $5B = 3L - 18$

$A = \frac{1}{2}LB = \frac{1}{2}L\frac{(3L - 18)}{5} = \frac{1}{10}(3L^2 - 18L)$

$= \frac{3}{10}(L^2 - 6L) = \frac{3L}{10}(L - 6)$

Min $\frac{dA}{dL} = 0 \Rightarrow 2L - 6 = 0 \Rightarrow L = 3$

$\Rightarrow A_{min} = \frac{3}{10}(9 - 18) = -2.7$

Value of $B = \frac{3L - 18}{5} = \frac{-9}{5} = -1.8$

10 $C = \pi f(r) = \pi f(30 - 3f) = \pi(30f - 3f^2)$

$\frac{dC}{df} = 0 \Rightarrow 30 - 6f = 0 \Rightarrow f = 5$

$r = 30 - 3(5) = 15$

Max. Value $= \pi \times 5 \times 15 = 75\pi$

when $f = 5$ and $r = 15$

11 $X = 2(10 + b)b = 20b + 2b^2$

$\frac{dX}{db} = 0 \Rightarrow 20 + 4b = 0 \Rightarrow b = -5$

Min. value of $X = 2 \times 5 \times -5 = -50$

12 $y := tx = t(12 - 2t) = 12t - 2t^2$

$\frac{dy}{dt} = 0 \Rightarrow 12 - 4t = 0 \Rightarrow t = 3$

$\therefore y = 3(12 - 6) = 18$ and this will be a max (positive t^2 quadratic)

13 $A = 2xy = 2(30 - 3y)y = 60y - 6y^2$

$\frac{dA}{dy} = 0 \Rightarrow 60 - 12y = 0 \Rightarrow y = 5$

giving $A = 2 \times 15 \times 5 = 150$ Max

14 $y = 3LM = 3(2M - 28)M = 6M^2 - 84M$

$\frac{dy}{dM} = 12M - 84 = 0 \Rightarrow M = \frac{84}{12} = 7$

giving $y = 3 \times -14 \times 7$

$= -294$ Min

15 $y = c^2 + g^2 = (8 - g)^2 + g^2 = 64 - 16g + 2g^2$

$\frac{dy}{dg} = -16 + 4g = 0$ when $g = 4$

$c = 8 - g = 8 - 4 = 4$

$\therefore$ Min. value of $y = 4^2 + 4^2 = 32$

16 $x + y = 6$

$S = x^2 + y^2 = x^2 + (6 - x)^2 = 36 - 12x + 2x^2$

$\frac{dS}{dx} = 0 \Rightarrow -12 + 4x = 0 \Rightarrow x = 3$

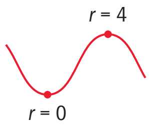
$y = 6 - 3 = 3$

$\therefore$ So $x = 3$ and $y = 3$

17 $y = r^2h = r^2(6 - r) = 6r^2 - r^3$

$\frac{dy}{dr} = 12r - 3r^2 = 0 \Rightarrow 3r(4 - r) = 0$

$\Rightarrow r = 0$ or 4



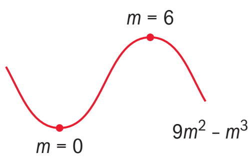
Max. y occurs at $r = 4$, giving

$y_{max} = 16(6 - 4) = 32$

18 $y = m^2n = m^2(9 - m) = 9m^2 - m^3$

$\frac{dy}{dm} = 18m - 3m^2 = 3m(6 - m)$

$= 0$ when $m = 0$ or 6

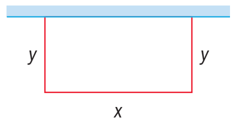


Min. at $m = 0$ giving $m^2n = 0$

Max. at $m = 6$ giving $m^2n = 36 \times 3 = 108$

Exercise 6N

1

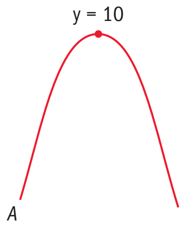


$x + 2y = 40$

Maximise area

$A = xy = (40 - 2y)y = 40y - 2y^2$

$\frac{dA}{dy} = 0 \Rightarrow 40 - 4y = 0 \Rightarrow y = 10$



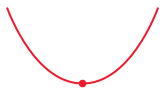
$A_{max} = 40 \times 10 - 200 = 200$ m^2 and wire should be bent with 2 sides of 10 m and 1 side of 20 m.

2 $y = 20 - x$

Minimise $S = 2x^2 + 3y^2 = 2x^2 + 3(20 - x)^2$

$= 2x^2 + 3(400 - 40x + x^2)$

$= 1200 - 120x + 5x^2$

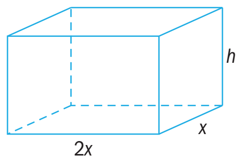


Stationary point will give min S

$\frac{dS}{dx} = 0 \Rightarrow -120 + 10x = 0$

$\Rightarrow x = 12$

3



Surface Area

$A = 2xh + 4xh + 2x^2$

$= 6xh + 2x^2$

$= 6xh + 2x^2$

Hence $6xh + 2x^2 = 150$

$\Rightarrow 3xh + x^2 = 75$

$\Rightarrow 3xh = 75 - x^2$

$\Rightarrow h = \frac{75 - x^2}{3x}$

Hence Volume $V = 2x \times x \times h$

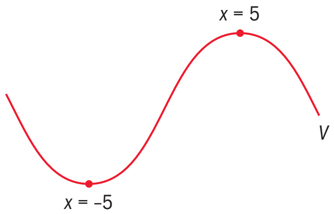
$= 2x^2 \frac{(75 - x^2)}{3x}$

$= \frac{2}{3}(75x - x^3)$

$\frac{dV}{dx} = 0 \Rightarrow 75 - 3x^2 = 0$

$\Rightarrow x^2 = 25$

$\Rightarrow x = 5$ (negative 5 is impossible)



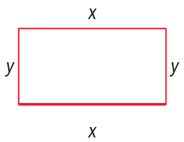
Hence $V_{max} = \frac{2}{3} \times 5(75 - 25)$

$= \frac{2}{3} \times 5 \times 50$

$= \frac{500}{3}$ cm^3

Width, length, and height is $5 \times 10 \times \frac{10}{3}$ cm

4



$3x + 2y = 24$

$\Rightarrow y = \frac{24 - 3x}{2}$

Maximise $A = xy = \frac{24x - 3x^2}{2}$

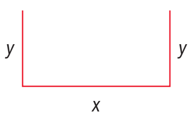
$\frac{dA}{dx} = 0 \Rightarrow 24 - 6x = 0 \Rightarrow x = 4$

$y = \frac{24 - 3(4)}{2} = 6$

Will give a maximum, as a "negative x^2 parabola"

Dimensions are 4×6 cm

5



$x + 2y = 120$

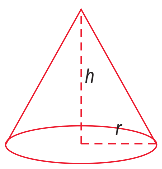
Maximise $A = xy = (120 - 2y)y = 120y - 2y^2$

$\frac{dA}{dy} = 0 \Rightarrow 120 - 4y = 0 \Rightarrow y = 30$

Will give maximum A since negative y^2 parabola

Width $= x = 120 - 60 = 60$ cm

6



$r + h = 12$

Maximise $V = \frac{1}{3}\pi r^2 h$

$= \frac{1}{3}\pi r^2(12 - r)$

$= \frac{1}{3}\pi(12r^2 - r^3)$

$\frac{dV}{dr} = 0 \Rightarrow 24r - 3r^2 = 0$

$\Rightarrow (8 - r)r = 0$

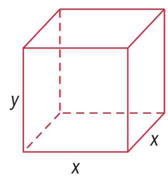
$\Rightarrow r = 0$ or $r = 8$

$r = 0$ gives min volume

$\therefore V_{max} = \frac{1}{3}\pi \times 64 \times 4 = \frac{256\pi}{3}$ cm^3

when $r = 8$ cm and $h = 4$ cm

7



$2x^2 + 4xy = 600$

$\Rightarrow x^2 + 2xy = 300$

Maximise $V = x^2 y$

$= x^2 \frac{(300 - x^2)}{2x}$

i.e. $V = \frac{1}{2}(300x - x^3)$

$\frac{dV}{dx} = 0 \Rightarrow 300 - 3x^2 = 0$

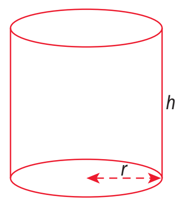
$\Rightarrow x^2 = 100$

$\Rightarrow x = \pm 10$ (-10 impossible and gives min)

$\therefore V_{max} = \frac{1}{2} \times 10(300 - 100)$

$= 1000$ cm^3

8



$2\pi r^2 + 2\pi rh = 600$

$\Rightarrow 2\pi rh = 600 - 2\pi r^2$

$\Rightarrow h = \frac{300 - \pi r^2}{\pi r}$

Maximise $V = \pi r^2 h = \frac{\pi r^2(300 - \pi r^2)}{\pi r}$

$= 300r - \pi r^3$

$\frac{dV}{dr} = 0 \Rightarrow 300 - 3\pi r^2 = 0$

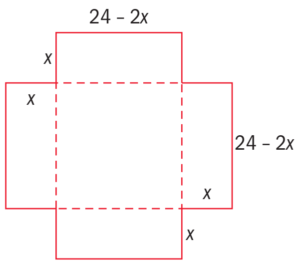
$\Rightarrow r^2 = \frac{100}{\pi}$

Negative r impossible and will give minimum so $r = \frac{10}{\sqrt{\pi}}$ for maximum V.

So dimensions are $r = \frac{10}{\sqrt{\pi}}$ and $h = \frac{200}{10\sqrt{\pi}} = \frac{20}{\sqrt{\pi}}$

$r \approx 5.64$ cm $\quad h \approx 11.28$ cm

9



$V = (24 - 2x)^2x$

$= 2^2(12 - x)^2x$

$= 4x(144 - 24x + x^2)$

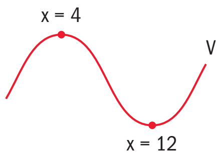
$\therefore\ V = 4\,(144x - 24x^2 + x^3),$

$\frac{dV}{dx} = 0 \Rightarrow 144 - 48x + 3x^2 = 0$

$\Rightarrow x^2 - 16x + 48 = 0$

$\Rightarrow (x - 12)\,(x - 4) = 0$

V_{max} at $x = 4$



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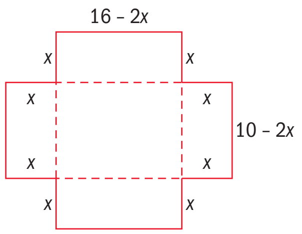


$V = (16 - 2x)\,(10 - 2x)x$

$= 2(8 - x)\,(5 - x)x$

$= 4x(8 - x)\,(5 - x)\ \text{cm}^3$

$V = 4x(40 - 13x + x^2)$

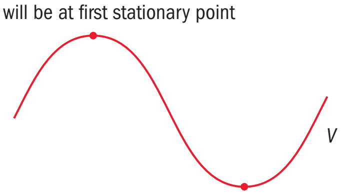
$= 4(x^3 - 13x^2 + 40x)$

$\frac{dV}{dx} = 0 \Rightarrow 3x^2 - 26x + 40 = 0$

$\Rightarrow (3x - 20)\,(x - 2) = 0$

$\Rightarrow x = \frac{20}{3}$ or $x = 2$

Max V will be at first stationary point

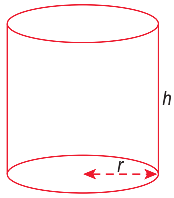


$x = 2$ give maximum.

So $V_{max} = 4(8 - 13 \times 4 + 80)$

$= 144\ \text{cm}^3$

11



$V = 350$

$\Rightarrow \pi r^2 h = 350$

a $r = 5 \Rightarrow h = \frac{350}{25\pi} = \frac{14}{\pi} \approx 4.46$ cm

b $r = 2 \Rightarrow h = \frac{350}{4\pi} = \frac{87.5}{\pi} \approx 27.85$ cm

c **i** $\pi r^2 h = 350$

ii $\Rightarrow h = \frac{350}{\pi r^2}$

iii $A = 2\pi r^2 + 2\pi rh$

$= 2\pi r^2 + 2\pi r \times \frac{350}{\pi r^2}$

$= 2\pi r^2 + \frac{700}{r}$

iv Minimise A

$\frac{dA}{dr} = 4\pi r - \frac{700}{r^2} = 0$

$\Rightarrow r^3 = \frac{700}{4\pi}$

$\Rightarrow r = \sqrt[3]{\frac{700}{4\pi}} = \sqrt[3]{\frac{175}{\pi}}$

Does give minimum A (check $\frac{dA}{dr}$ either side).

So $r \approx 3.82$ cm

and $h \approx 7.64$ cm

v $A_{min} = 274.9\ \text{cm}^2$

12 **a** 250 m

b Length $= 2L + 3W = 400 + 750 = 1150$ m.

c $LW = 50000$

d Length $y = 2L + 3W$

$= \frac{100\,000}{W} + 3W$

$\frac{dy}{dW} = 0 \Rightarrow 3 - \frac{100\,000}{W^2} = 0$

$\Rightarrow W^2 = \frac{100\,000}{3}$

$\Rightarrow W = 182.6$ m

Will give a minimum (check $\frac{dy}{dW}$ either side)

This gives $L = \frac{50000}{W} \approx 273.9$ m

Perimeter $= 2L + 2W \approx 913$ m

13 a $(2L + 2W) \times 3 + 5W = \$3950$

b $LW = 50\,000$

c Cost, $C = 6L + 6W + 5W$

$= 6L + 11W$

$= \frac{300\,000}{W} + 11W$

$\frac{dC}{dW} = \frac{-300\,000}{W^2} + 11$

$= 0$ when $W^2 = \frac{300\,000}{11}$

i.e. $W \approx 165.1$ m

Gives minimum, by checking $\frac{dC}{dW}$ either side.

Field is 165.1×302.8 m

$C_{min} = \frac{300\,000}{165.1} + 11 \times 165.1 = \$3633.18$

14 a $h = 16$ cm

Page is 22 cm × 13 cm

Area is 286 cm^2

b $293\frac{1}{7}$ cm^2

c $A = wh$

d $P = (w + 4)(h + 6)$

e $P = wh + 4h + 6w + 24 = 144 + 24 + 4h + \frac{6 \times 144}{h}$

$= 168 + 4h + \frac{86}{h}$

$\frac{dP}{dh} = 4 - \frac{864}{h^2}$

$\Rightarrow h^2 = 216$

$\Rightarrow h \approx 14.7$ cm

giving $w = 9.8$ cm

Minimum size of Full page is 20.7 cm × 13.8 cm (gives minimum by checking either side)

15 $2w^2h = 225\,000$

a i 50 cm

ii $\Rightarrow 2 \times 5\not{0} \times 5\not{0}\,h = 225\,0\not{0}\not{0}$

$h = \frac{2250}{50} = 45$ cm

iii length $= 4w + 2w + 4h$

$= 6w + 4h$

$= 480$ cm

b $2x^2h$

c $L = 6w + 4h$

$= 6x + 4h = 6x + \overset{2}{\not{4}} \times \frac{225\,000}{\not{2}x^2}$

$= 6x + \frac{450\,000}{x^2}$

d $\frac{dL}{dx} = 6 - \frac{900\,00}{x^3}$

$= 0$ when

$x^3 = \frac{900\,000}{6} = 150\,000$

$\Rightarrow x = 53.1$ cm (will give minimum – check gradient either side)

Dimensions are width 53.1 cm

length 106.2 cm

height 39.8 cm

Length of frame ≈ 478.2 cm

7 Number and algebra 2

Answers

Skills check

1 $A = \pi r^2 + \pi rs$

a $A = \pi(4)^2 + \pi(4)(3) = 88.0$

b $A - \pi r^2 = \pi rs$

$s = \frac{A - \pi r^2}{\pi r}$

2 **a** $630 \times 1.04 = 655.20$ GBP

b $652 \times 1.12 = 730.24$

c $120 \times 0.80 = € 96$

3 $x - 2y = 11$ (1)

$3x + y = -2$ (2)

(2) × 2 $6x + 7y = -4$ (3)

(1) + (3) $7x = 7$

$x = 1$

$y = -5$

Exercise 7A

1 **a** $u_8 = 3 + (8 - 1)4 = 3 + 28 = 31$

b $u_{150} = 3 + (150 - 1)4 = 3 + 596 = 599$

2 **a** $u_1 + 2d = 8$ and $u_1 + 8d = 26$

b using the GDC $u_1 = 2$ and $d = 3$

3 $u_1 = -12$

$u_1 + 8d = 16$

$-12 + 8d = 16$

$8d = 28$

$d = 3.5$

4 **a** $u_n = u_1 + (n - 1)\,d = 3 + (n - 1)4$

$= 3 + 4n - 4 = 4n - 1$

b $u_{50} = 4(50) - 1 = 199$

5 **a** $u_1 = 42 - 3(1) = 39$

$u_2 = 42 - 3(2) = 36$

b $42 - 3n = -9$

$-3n = -9 - 42 = -51$

$n = \frac{-51}{-3} = 17$

c $u_k + u_{k+1} = u_1 + (k - 1)d + u_1 + (k + 1 - 1)d$

$= 39 + (k - 1)(-3) + 39 + (k)(-3)$

$= 78 - 3k + 3 - 3k = 81 - 6k$

$= 33$

$-6k = 33 - 81 = -48$

$k = 8$

6 **a** $u_6 = u_1 + (6 - 1)d = 34$

$d = 6$

$u_1 + 5(6) = 34$

$u_1 = 34 - 30 = 4$

b $u_n = 4 + (n - 1)(6) = 316$

$4 + 6n - 6 = 316$

$6n = 318$

$n = 53$

7 $u_1 = 8$ $d = 7$

$u_n = 8 + (n - 1)(7) = 393$

$8 + 7n - 7 = 393$

$7n = 392$

$n = 56$

8 **a** $d = -1 - (-5) = -1 + 5 = 4$

b $u_{13} = -5 + 12(4) = -5 + 48 = 43$

c $u_n = -5 + (n - 1)(4) = 75$

$-5 + 4n - 4 = 75$

$4n = 84$

$n = 21$

9 **a** $d = 10.5 - 8 = 2.5$

b $u_{12} = 8 + 11(2.5) = 35.5$

c $u_n = 8 + (n - 1)(2.5) = 188$

$8 + 2.5n - 2.5 = 188$

$2.5n = 182.5$

$n = 73$

10 **a** $u_1 = 12 + 7(1) = 19$

$u_2 = 12 + 7(2) = 26$

b $26 - 19 = 7 = d$

c $u_{25} = 19 + 24(7) = 187$

Exercise 7B

1 **a** 26

b $u_{50} = 1 + 49(5) = 246$

c $S_{50} = \frac{50}{2}(2 \times 1 + 49 \times 5) = 6175$

2 **a** $(5k + 2) - (k + 4) = (10k - 2) - (5k + 2)$

$4k - 2 = 5k - 4$

$k = 2$

b $2 + 4 = 6$

$5(2) + 2 = 12$

$10(2) - 2 = 18$

c $d = 12 - 6 = 6$

d $u_{25} = 6 + 24(6) = 150$

e $S_{25} = \frac{25}{2}(2 \times 6 + 24 \times 6) = 1950$

3 **a** **i** $u_6 = u_1 + 5d = 20$
$u_{11} = u_1 + 10d = 50$
Using GDC, $d = 6$

ii $u_1 = -10$,

b $S_{100} = \frac{100}{2}(2 \times -10 + 99 \times 6) = 28\,700$

4 **a** $u_n = 12 + (n-1)(-4)$
$= 12 - 4n + 4 = 16 - 4n$

b $S_{80} = \frac{80}{2}(2 \times 12 + 79 \times -4)$
$= -11\,680$

5 **a** **i** $u_1 + d = 2$ and $u_9 = u_1 + 8d = -19$
$7d = -21$
$d = -3$

ii $u_1 = 5$

b $S_{60} = \frac{60}{2}(2 \times 5 + 59 \times -3) = -5010$

6 $u_n = u_1 + (n-1)d = -7 + (n-1)5$
$= -7 + 5n - 5 = 238$
$5n = 250$
$n = 50$
$S_{50} = \frac{50}{2}(2 \times -7 + 49 \times 5) = 5775$

7 $u_n = u_1 + (n-1)d = 26 + (n-1)(-1.5)$
$= 26 - 1.5n + 1.5 = -17.5$
$-1.5n = -45$
$n = 30$
$S_{30} = \frac{30}{2}(2 \times 26 + 29 \times -1.5) = 127.5$

8 **a** $6k - (3k+4) = (3k+4) - (4k-2)$
$3k - 4 = -k + 6$
$4k = 10$
$k = 2.5$

b $4(2.5) - 2 = 8$
$3(2.5) + 4 = 11.5$
$6(2.5) = 15$

c $d = 11.5 - 8 = 3.5$

d $u_{15} = 8 + 14(3.5) = 57$

e $S_{15} = \frac{15}{2}(2 \times 8 + 14 \times 3.5) = 487.5$

Exercise 7C

1 **a** $u_{18} = 50 + 17(25) = 475$

b $S_{18} = \frac{18}{2}(2 \times 50 + 17 \times 250) = 4725$

2 **a** 2.5 minutes = 2.5 × 60 seconds = 150 seconds
$u_3 = 150 + 2 \times 10 = 170$ seconds
= 2 minutes 50 seconds

b $S_{10} = \frac{10}{2}(2 \times 150 + 9 \times 10)$
= 1950 seconds = 32 minutes 30 seconds

3 $u_1 = a$, $d = p$
$u_6 = 2 \times u_3$
$a + 5p = 2(a + 2p)$
$a + 5p = 2a + 4p$
$p = a$
$u_{10} = a + 9p = 4000$
$a + 9a = 4000$
$a = 400$
$p = 400$

4 **a** $u_{10} = 150 + 9 \times 250 = 2400$

b $S_{10} = \frac{10}{2}(2 \times 150 + 9 \times 250) = 12750$

c Option A gets 1200 × 10 = 12000
Therefore Option B gives 750 more than Option A

5 **a** $u_{10} = 100 + 9 \times 10 = 190$

b $S_{15} = \frac{15}{2}(2 \times 100 + 14 \times 10) = 2550$

6 **a** $u_{10} = 18 + 9 \times 2 = 36$

b $S_{25} = \frac{25}{2}(2 \times 18 + 24 \times 2) = 1050$

Exercise 7D

1 **a** $r = \frac{8}{4} = 2$ **b** $u_{20} = 4(2)^{19} = 2\,097\,152$

2 **a** $r = \frac{2}{6} = \frac{1}{3}$ **b** $u_{10} = 6\left(\frac{1}{3}\right)^9 = 0.000\,305 = \frac{2}{6561}$

3 **a** $r = -\frac{640}{1280} = -0.5$ **b** $u_8 = 1280(-0.5)^7 = -10$

4 **a** $u_1 = 5$ $u_3 = 5r^2 = 20$
$r^2 = \frac{20}{5} = 4$
$r = 2$

b $u_7 = 5(2)^6 = 320$

5 **a** $u_2 = u_1 r = 18$ $u_4 = u_1 r^3 = \frac{81}{2}$
$\frac{u_1 r^3}{u_1 r} = \frac{81}{2} \times \frac{1}{18} = 2.25$
$r^2 = 2.25$
$r = 1.5$

b $u_1 = \frac{18}{r} = \frac{18}{1.5} = 12$
$u_8 = 12(1.5)^7 = 205.031\,25$

6 **a** $a = -16 \times \frac{1}{2} = -8$

b $u_8 = -16\left(\frac{1}{2}\right)^7 = -0.125$

7 $u_2 = u_1 r = 18$ $u_4 = u_1 r^3 = 8$
$\frac{u_1 r^3}{u_1 r} = \frac{8}{18} = \frac{4}{9}$
$r^2 = \frac{4}{9}$
$r = \frac{2}{3}$

8 **a** $u_1 = 12\ u_3 = u_1r^2 = 48$

$12r^2 = 48$

$r^2 = \frac{48}{12} = 4$

$r = 2$

b $u_{12} = 12(2)^{11} = 24\,576$

Exercise 7E

1 **a** $p = 8 \times 0.5 = 4$

b $u_7 = 16(0.5)^6 = 0.25$

c $S_{15} = \frac{16(1-0.5^{15})}{1-0.5} = 32.0\ (3\text{sf})$

2 **a** $u_1 = 2;\ u_3 = u_1r^2 = 2r^2 = 32$

$r^2 = 16$

$r = \pm 4$

b For $r = 4$, $S_{10} = \frac{2(1-4^{10})}{1-4} = 699\,050$

For $r = -4$, $S_{10} = \frac{2(1-(-4)^{10})}{1-(-4)} = -419\,430$

3 **a** $r = \frac{6}{-2} = -3$

b $S_{10} = \frac{-2(1-(-3)^{10})}{1-(-3)} = 29524$

4 $u_2 = u_1r = 21$

$u_4 = u_1r^3 = 5.25$

a $\frac{u_1 r^3}{u_1 r} = \frac{5.25}{21} = 0.25$

$r^2 = 0.25$

$r = \pm 0.5$

b For $r = 0.5$, $u_1 = \frac{21}{0.5} = 42$

$S_{10} = \frac{42(1-0.5^{10})}{1-0.5} = 83.9$

For $r = -0.5$, $u_1 = \frac{21}{-0.5} = -42$

$S_{10} = \frac{-42(1-[0.5)]^{10})}{1-(-0.5)} = -28.0$

5 $u_1r^{(n-1)} = 8192$

$2(2)^{(n-1)} = 8192$

$2^{(n-1)} = \frac{8192}{2} = 4096$

Using GDC, $n = 13$

$S_{13} = \frac{2(1-2^{13})}{1-2} = 16382$

6 $u_1 = -96\ r = \frac{48}{-96} = -0.5$

$u_n = -96(-0.5)^{n-1} = \frac{-3}{8}$

$(-0.5)^{n-1} = \frac{\left(\frac{3}{8}\right)}{-96} = \frac{1}{256} = 0.00390625$

Using GDC, $n = 9$

$S_9 = \frac{-96(1-(-0.5)^9)}{1-(-0.5)} = -64.125$

Exercise 7F

1 If it grows 2% each week then we multiply by 1.02 each time.

So, after 10 weeks the plant is $0.8\,(1.02)^9 = 0.956$ m

2 Multiplying factor is 0.92. So, after 5 years the car is worth $75\,000 \times 0.92^4 = 53\,729.47$ GBP

3 $u_1 = 10, r = 2$

$S_{10} = \frac{10(1-2^{10})}{1-2} = 10230$ BGN

4 **a** $u_1 = 80$ and $r = 1.05$

$u_8 = 80(1.05)^7 = 112.57$ Dinar

b $S_{12} = \frac{80(1-1.05^{12})}{1-1.05} = 1237.37$ Dinar

5 $r = 1.04,\ u_1 = 210000$

In 2013, the population is $210\,000(1.04)^3 = 236\,221$

6 **a** $140\,000\ r^2 = 145\,656$

$r^2 = \frac{145\,656}{140\,000} = 1.0404$

$r = 1.02$

so, population at end of 2007 $= 140\,000(1.02) = 142\,800$

b At end of 2012 population $= 140\,000(1.02)^6 = 157\,663$

7 **a** $r = \frac{6300}{6000} = 1.05$

b $S_6 = \frac{6000(1-1.05^6)}{1-1.05} = \$40811.48$

8 **a** $u_1 = 8$ and $\frac{24}{8} = 3$ and $\frac{72}{24} = 3$ So $r = 3$

b $u_5 = 8(3)^4 = 648$

c $S_7 = \frac{8(1-3^7)}{1-3} = 8744$

Exercise 7G

1 $\frac{3500}{0.3236} = 10\,815.82$ ringgits

2 **a** $500 \times 0.783 = 391.50$ euros

b $391.50 - 328 = 63.50$ euros

$\frac{63.50}{1.172} = 54.18$ GBP

3 **a** $800 \times 0.758 = 606.40$ euros

b $\frac{606.40}{0.835} = 726.23$ CAD

c $800 - 726.23 = 73.77$ CAD

4 **a** $8000 \times 0.111 = 888$ euros

b $\frac{888}{0.121} = 7338.84$ SEK

c $8000 - 7338.84 = 661.16$ SEK

5 **a** $500 \times 3.984 = 1992$ ZAR

b $\frac{500}{3.984} = 125.50$ BRL

6 **a** 250 − 4 = 246 GBP
246 × 1.173 = 288.56 euros

b 2.25 × 10 = 22.5 euros per kg
$\frac{22.5}{1.173}$ = 19.18 GBP

7 **a** 2500 × 1.319 = 3297.50 USD

b 3297.50 − 2050 = 1247.50 USD
$\frac{1247.50}{1.328}$ = 939.38 EUR

c $1247.50 \times \frac{0.6}{100}$ = 7.485 USD commission
She changes 1247.50 − 7.485
= 1240.015 USD
$\frac{1247.50}{1.261}$ = 983.36 EUR
983.36 − 939.38 = 3.98 EUR was lost by changing in the US

8 **a** 2550 × 0.080 86 = 206 yuan

b $\frac{2150}{0.01231}$ = 174 655 yen

c 1 JPY = 0.009261 EUR = 0.007897 GBP
So, 1 EUR = $\frac{0.007\,897}{0.009\,261}$ = 0.85 GBP

9 **a** $3000 \times \frac{1.5}{100}$ = 45 EUR

b 3000 − 45 = 2955 EUR
2955 × 0.8524 = 2518.84 GBP

c 2518.84 − 2100 = 418.84 GBP
418.84 × 1.161 = 486.27 EUR

10 **a** 500 × 44.95 = 22 475 IDR

b $500 \times \frac{2}{100}$ = 10 USD commission
Jose exchanges 500 − 10 = 490 USD
490 × 468.9 = 229 761 CLP

11 **a** 1 USD = 0.759 EUR so,
1 EUR = $\frac{1}{0.759}$ = 1.3175 = p
1 JPY = 0.00926 EUR so,
1 EUR = $\frac{1}{0.00926}$ = 107.99 = q

b **i** 1 EUR = 0.852 GBP so
150 GBP = $\frac{1}{0.852} \times 150$ = 176.06 EUR

ii 2.4% of 150 = $\frac{2.4}{100} \times 150$ = 3.60
150 − 3.60 = 146.40 GBP

12 **a** $3000 \times \frac{2.5}{100}$ = 75 USD commission
She exchanges 3000 − 75 = 2925 USD
2925 × 0.652 = 1907.10 GBP

b 550 × 1.18 = 649 EUR
She only gets 629 EUR, so 649 − 629 = 20 EUR is the commission.
$\frac{20}{1.18}$ = 16.95 GBP commission

Exercise 7H

1 **a** $3000 \times (1.065)^{15} = 7715.52$ JPY

b $3000 \times (1.065)^n = 6000$, using GDC, $n = 11$ years

2 **a** Andrew has 3105.94 euros
Billy has 3090.54 euros
Colin has 3067.47 euros

b 9.21 years **c** 16.2 years

3 **a** \$6110.73 **b** $r = 3.79$

4 **a** 23348.48 EGP **b** 22.4 years

5 **a** 61252.49 SGD **b** 75070.16 SGD

6 Mr Lin has 11698.59 CNY
Mr Lee has 11707.24 CNY
Mr Lee has earned most interest

7 **a** 1348.85 GBP

b 2965 GBP

c 11.6 years

8 **a** $(1 + \frac{6}{100})^1 + (8000 - a)(1 + \frac{5}{100})^1 = 8430$

b $1.06a + (8000 - a)1.05 = 8430$
$1.06a + 8400 - 1.05a = 8430$, $0.01a = 30$,
$a = 3000$
3000 euros in Bank A, 5000 euros in Bank B

Exercise 7I

1 If inflation is 2.3% then the multiplying factor is $1 + \frac{2.3}{100} = 1.023$
In 2013 a bag of potatoes will cost
$3.45 \times 1.023^3 = 3.69$ euros

2 The multiplying factor is $1 + \frac{3.2}{100} = 1.032$
After 5 years the house is worth
$3\,200\,000 \times 1.032^5 = 3\,745\,833$ MXN

3 The car depreciates so the multiplying factor is

$1-\frac{8}{100}=0.92$

After 4 years it is worth $12\,300\times 0.92^4$ = 8811.63 USD

4 Price increases so multiplying factor is

$1+\frac{2.03}{100}=1.0203$

After 6 years the gold is worth $45\times 1.0203^6=50.77$ CAD

5 Shares depreciate so multiplying factor

$1-\frac{15}{100}=0.85$

After 2 years the shares are worth $18.95\times 0.85^2=13.69$ KRW per share.

6 Price increases so multiplying factor is

$1+\frac{1.8}{100}=1.018$

After 10 years the vase is worth $24\,000\times 1.018^{10}=28\,687.26$ GBP

7 The price depreciates so the multiplying factor is

$1-\frac{4.2}{100}=0.958$

After 8 years the yacht is worth $85\,000\times 0.958^8=60303.57$ USD

8 The rate increases so the multiplying factor is

$1+\frac{3.1}{100}=1.031$

After 5 years she should insure the contents for $103\,000\times 1.031^5=119\,985.99$ euros.

Review exercise

Paper 1 style questions

1 **a** Let x be interest rate.
Then $500\times x^2=625$

$x^2=\frac{625}{500}$

$x\approx 11.8\%$

b $500\ (1.118)^n=1000$
using GDC, $n=6.21$ or 7 years

2 **a** Increase in price so multiplying factor is

$1+\frac{2.3}{100}=1.023$

$240\,000\times 1.023^3=256\,943.80$ USD

b $200\,000\times r^3=214\,245$

$r^3=\frac{214\,245}{200\,000}=1.071225$

$r=\sqrt[3]{1.071225}=1.0232$

So the percentage increase is 2.32%

3 **a** $1200\times(1.043)^4=1420.10$ GBP
So Joseph earns 220.10 GBP interest

b $1200\times(1.043)^n=1450$
using GDC, $n=4.7$ years
$\therefore$ must invest for 5 years.

c $1200\times(1.043)^n=2400$
using GDC, $n=16.5$ or 17 years.

4 **a** $125\times 0.753=94.125$ EUR

b 610 EUR = 800 AUD, so

$1\text{ EUR}=\frac{800}{610}=1.3115\text{ AUD}$

0.753 EUR = 1 USD so,

$1\text{ EUR}=\frac{1}{0.753}=1.328\text{ USD}$

1.328 USD = 1.3115 AUD so,

$1\text{ USD}=\frac{1.3115}{1.328}=0.988\text{ AUD}$

5 **a** Increase is 3.5% so multiplying factor is

$1+=\frac{3.5}{100}=1.035$

In 2012 the fees will be 1500×1.035^2 = 1607 GBP

b The total fees for 5 years,

$S_5=\frac{1500(1-1.035^5)}{(1-1.035)}=8043.70\text{ GBP}$

6 **a** Use formula for compounding quarterly:

$18000\left(1+\frac{4.5}{4\times 100}\right)^{(4\times 15)}$

$=35219.61=€35220$

b Use Finance Solver on GDC: 19862.21

$=18000\left(1+\frac{4.5}{4\times 100}\right)^{(4\times n)}$

$\Rightarrow n=26.4$ or 27 months

7 **a** $u_1+3d=15$ and $u_1+9d=33$
Subtracting, $6d=33-15=18$ So $d=3$ and $u_1=6$

b $u_{50}=6+49\times 3=153$

c $S_{50}=\frac{50}{2}\ (2\times 6+49\times 3)=3975$

8 $u_n=-15+(n-1)2=-15+2n-2=27$

$2n=27+15+2=44$

$n=22$

$S_{22}=\frac{22}{2}\ (2\times -15+21\times 2)=132$

9 a $u_1r = 30$ and $u_1r^3 = 120$

$\frac{u_1r^3}{u_1r} = r^2 = \frac{120}{30} = 4$ so $r = \pm 2$

And $u_1r = 30$, so

i when $r = 2$, $u_1 = \frac{30}{2} = 15$

ii when $r = -2$, $u_1 = \frac{30}{-2} = 15$

so $u_1 = \pm 15$

b $u_6 = 15(2)^5 = 480$ or $-15(-2)^5 = 480$

c $S_8 = \frac{15(2^8-1)}{(2-1)} = 3825$ or $\frac{-15((-2)^8-1)}{(-2-1)} = 1275$

10 a $r = \frac{18}{54} = \frac{1}{3}$

b $u_7 = 54\times\left(\frac{1}{3}\right)^6 = 0.0741 = \frac{2}{27}$

c $S_{10} = \frac{54\left(1-\left(\frac{1}{3}\right)^{10}\right)}{\left(1-\frac{1}{3}\right)} = 81$

11 a $u_1r = -4$ and $u_1r^3 = -1$

$\frac{u_1r^3}{u_1r} = r^2 = \frac{-1}{-4} = 0.25$

So, $r = \pm 0.5$ and $u_1 = \pm 8$

b $u_6 = 8(0.5)^5 = 0.25$ or $-8(-0.5)^5 = 0.25$

c $S_6 = \frac{8(0.5^6-1)}{(0.5-1)} = 15.75$ or $\frac{-8((-0.5)^6-1)}{(-0.5-1)} = -5.25$

12 a $200 + 10 \times 25 = 450$

b $200 \times 1.15^{10} = 809.11$

c $200 \times 1.15^x > 200 + 25x$

Using GDC, $x = 3.21$

So, after 4 times John will have more money than Mary

13 a $u_{36} = 8 + 35 \times 8 = 288$

b $u_6 = 3r^5 = 8 + 11 \times 8 = 96$

c $r^5 = \frac{96}{3} = 32$

$r = \sqrt[5]{32} = 2$

Paper 2 style questions

1 a i $1000 + 7 \times 250 = 2750$ for Option 2

ii $15 \times 2^7 = 1920$ for option 3

b total for option 2 $= \frac{10}{2}(2 \times 1000 + 9 \times 250)$

$= 21250$

c Option 1 total $= 10 \times 2000 = 20000$

Option 3 total $= \frac{15(2^{10}-1)}{(2-1)} = 15345$

Option 2 has the greatest total value

2 a Choice A: $12 \times 150 = 1800$

Choice B: $1600\ \frac{(1+10)}{(1200)^{12}} = 1767.54$

Choice C: $\frac{12}{2}(2 \times 105 + 11 \times 10) = 1920$

Choice D: $120\ \frac{(1.05^{12}-1)}{(1{,}05-1)} = 1910.06$

b C, because it has the largest total.

c Using the GDC, $r = 6.27\%$

3 a i 2250 and 2500

ii $2000 + 19 \times 250 = 6750$

iii $\frac{20}{2}(2 \times 2000 + 19 \times 250) = 87500$

b i $2800 \times 1.05 = 2940$

ii $2800 \times 1.05^4 = 3403.42$

c $\frac{2800(1.05^{20}-1)}{(1.05-1)} = 92584.67$

$92584.67 - 87500 = 5084.67$ will be saved by choosing Option 1

4 a $6k + 4 - 5k = 5k - (3k + 1)$

$k + 4 = 2k - 1$

$5 = k$

b $3(5) + 1 = 16$, $5(5) = 25$, $6(5) + 4 = 34$

c $25 - 16 = 9$

d $u_{15} = 16 + 14 \times 9 = 142$

e $S_{20} = \frac{20}{2}(2 \times 16 + 19 \times 9) = 2030$

5 a $28000 \times 1.04^3 = 31496.19$

b i $24000 \times 1.05^x > 28000 \times 1.04^x$

Using the GDC, $x = 16.1$

So, in the 17th year his spending will be more than his salary.

ii $24000 \times 1.05^{17} - 28000 \times 1.04^{17}$

$= 467.23$

6 a $\frac{2(4^n-1)}{(4-1)} = 11\,184\,810$

Using GDC, $n = 12$

b $r = \frac{\left(\frac{2}{5}\right)}{2} = \frac{1}{5} = 0.2$

c $\frac{2(1-0.2^{10})}{(1-0.2)} = 2.5$

8 Sets and probability

Answers

Skills check

1 **a** 5 is an integer, real and rational $\left(\text{since it can be written as } \frac{5}{1}\right)$

b $1.875 = 1\frac{7}{8}$ is not an integer, but is both real and rational, since it can be written as $\frac{15}{8}$

c $0.333 = \frac{333}{1000}$ is not an integer, but is both real and rational. Note that $0.333 \neq \frac{1}{3}$

d 0.3030030003... is real, but not rational.

e $\sqrt{0.5625} = \frac{3}{4}$ is both real and rational.

f $\sqrt[3]{2.744} = 1.4 = \frac{7}{5}$ is both real and rational.

g π^2 is real, but not rational.

2 **a-d** −2, −1, 0, 1, 2, 3

3 **a** **i** 1, 2, 3, 4, 6, 12

ii 1, 2, 4, 8

iii 1, 17

iv 1, 5, 25

v 1, 2, 3, 4, 6, 8, 12, 24

b **i** 2, 3

ii 2

iii 17

iv 5

v 2, 3

c 17 is prime.

d zero has an infinite number of factors.
zero is an integer, it is rational and it is real, but it is not prime.

Exercise 8A

1 $M = \{2, 3, 4\}$ $n(A) = 3$
$N = \{1, 2, 3, 4, 5\}$ $n(B) = 5$
$P = \{1, 2, 3, 4, 5\}$ $n(C) = 5$
$S = \{(1, 4), (2, 3), (3, 2), (4, 1)\}$ $n(D) = 4$
$T = \{(0, 5), (1, 4), (2, 3), (3, 2), (4, 1), (5, 0)\}$
$n(E) = 6$

$V = \{ \}$ or $F = \varnothing$ $n(F) = 0$

$W = \{1, 2, 4, 5, 10, 20\}$ $n(G) = 6$

X is an infinite set and so elements cannot be listed. $n(H) = \infty$

2 **a** $\{4, 5, 6\}$ **b** $\{2, 4, 6\}$
c $\{7, 9, 11\}$ **d** $\{5, 9, 13, 17, 21\}$
e $\{(2, 2), (4, 4), (6, 6), (8, 8), (10, 10)\}$
f $\{(6, 3), (10, 5)\}$

Exercise 8B

1 $N \subseteq M$ False **2** $S \subseteq T$ True
3 $P \subseteq M$ False **4** $W \subseteq X$ True
5 $N \subseteq P$ True **6** $P \subseteq N$ True
7 $\varnothing \subseteq W$ True **8** $W \subseteq W$ True

2 **a** $\varnothing, \{a\}$
b $\varnothing, \{a\}, \{b\}, \{a, b\}$
c $\varnothing, \{a\}, \{b\}, \{c\}, \{a, b\}, \{a, c\}, \{b, c\}, \{a, b, c\}$
d There are 16 of these!
2^n
32
7

3 **a** There are none.
b $\{a\}, \{b\}$
c $\{a\}, \{b\}, \{c\}, \{a, b\}, \{a, c\}, \{b, c\}$
d There are 14 of these!
$2^n - 2$
62
8

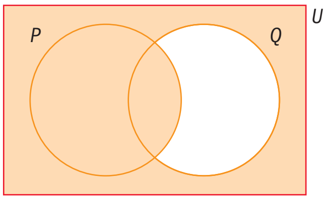
Exercise 8C

Consider the sets below.
$M = \{x \mid 2 \le x < 5, x \in \mathbb{Z}\}$
$N = \{x \mid 0 < x \le 5, x \in \mathbb{Z}\}$
$P = \{x \mid -2 \le x < 6, x \in \mathbb{Z}^+\}$
$S = \{(x, y) \mid x + y = 5, x \in \mathbb{Z}^+, y \in \mathbb{Z}^+\}$
$T = \{(x, y) \mid x + y = 5, x \in \mathbb{Z}, y \in \mathbb{Z}\}$
$V = \{p \mid p$ is a prime number and a multiple of $4\}$
$W = \{x \mid x$ is a factor of $20\}$
$X = \{x \mid x < 200, x \in \mathbb{R}\}$

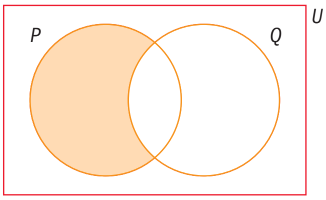
State whether the following are true or false:
a $N \subset M$ False **b** $S \subset T$ True
c $P \subset M$ False **d** $W \subset X$ True
e $M \subset P$ True **f** $P \subset N$ True
g $\varnothing \subset T$ False **h** $V \subset W$ False

Exercise 8D

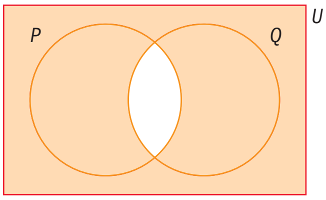
1 a $P \cup Q'$



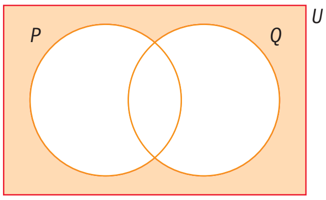
b $P \cap Q'$



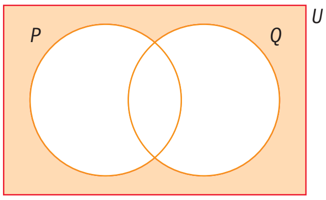
c $P' \cup Q'$



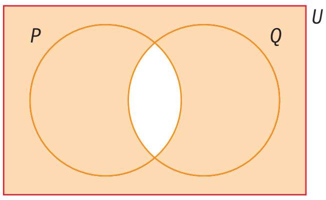
d $P' \cap Q'$



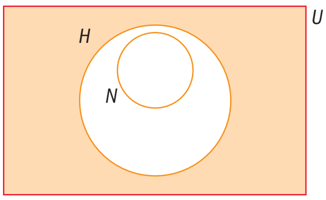
e $(P \cup Q)'$



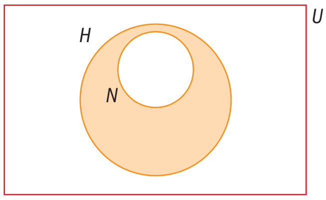
f $(P \cap Q)'$



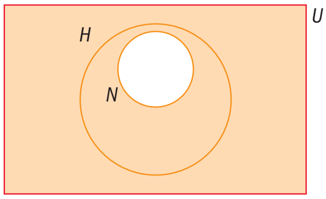
2 a H'



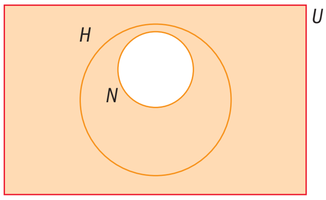
b $H \cap N'$



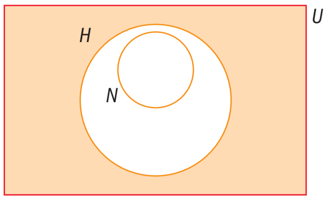
c N'



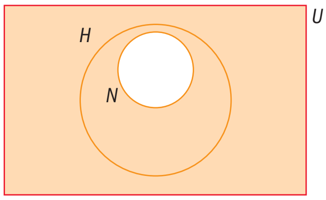
d $H' \cup N'$



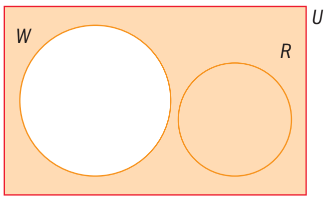
e $H \cap N'$



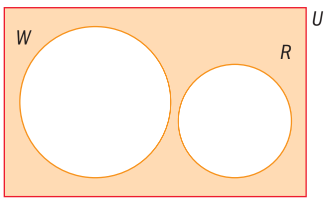
f $H \cup N'$



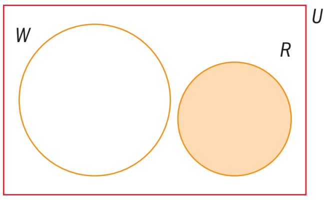
3 a W'



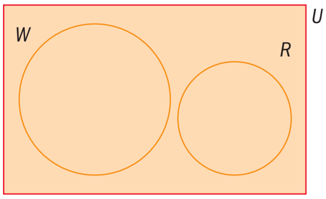
b $W' \cap R'$



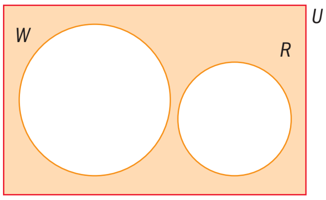
c $W' \cap R$



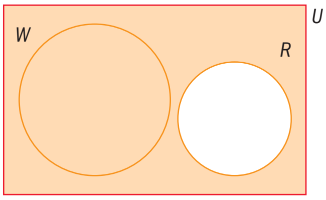
d $W' \cup R'$



e $(W \cup R)'$



f $(W' \cap R)'$



Exercise 8E

1 **a** $F \subset G$ False **b** $n(F \cup G) = 6$ True

c $n(G') = 8$ False **d** $n(F \cup H) = 6$ False

e $H \cup F = G'$ False **f** $F' \subset H$ False

g $n(G' \cap H) = 5$ False **h** $n(F' \cap G) = 5$ False

2 **a** U $\{b, c, d, e, f, g, h, k\}$ **b** R $\{b, d, e, f\}$

c R' $\{c, g, h, k\}$ **d** T $\{c, d, e, k\}$

e T' $\{b, f, g, h\}$

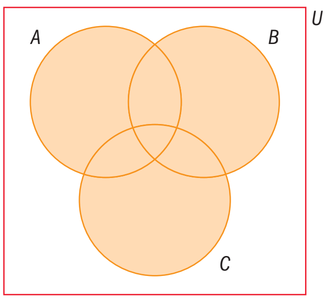
3 **a** A $\{q, t, x, w\}$ **b** A' $\{p, r\}$

c $A \cup B'$ $\{p, q, r, t, x, w\}$ **d** $A \cap B'$ $\{q, x, w\}$

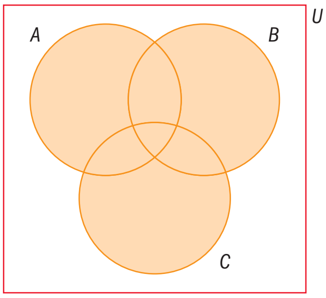
e $A' \cup B'$ $\{p, q, r, x, w\}$

Exercise 8F

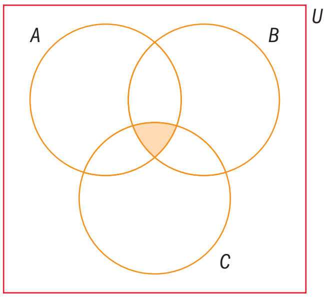
1 **a** **i** $(A \cup B) \cup C$



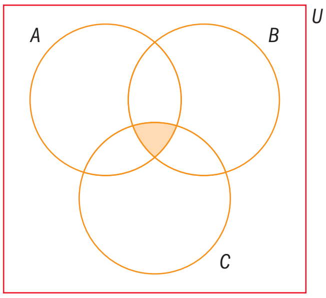
ii $A \cup (B \cup C)$



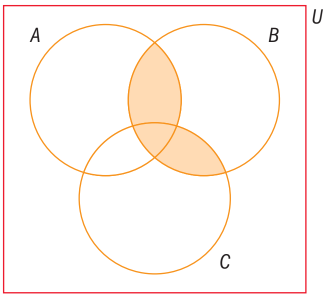
b **i** $(A \cap B) \cap C$



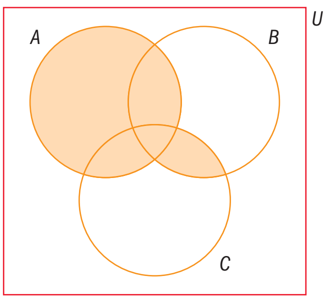
ii $A \cap (B \cap C)$



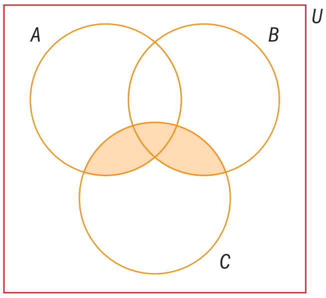
c **i** $(A \cup C) \cap B$



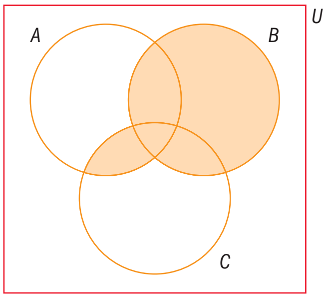
ii $A \cup (C \cap B)$



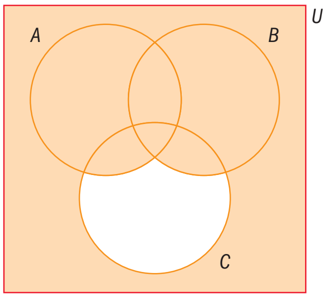
d **i** $C \cap (A \cup B)$



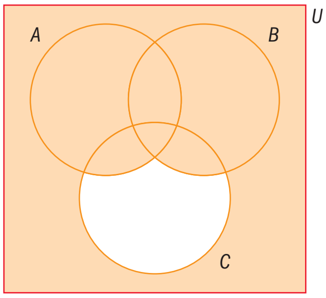
ii $B \cup (C \cap A)$



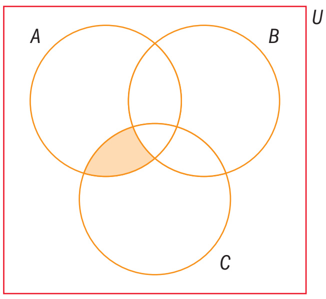
e **i** $(A \cup B) \cup C'$



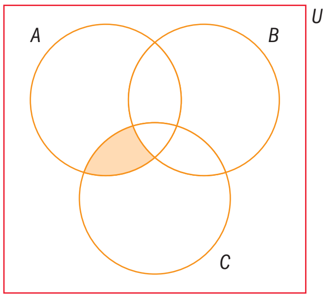
ii $A \cup (B \cup C')$



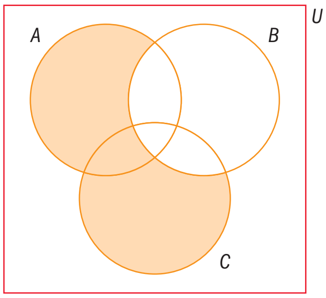
f **i** $(A \cap B') \cap C$



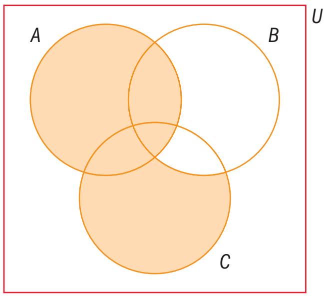
ii $A \cap (B' \cap C)$



g **i** $(A \cup C) \cap B'$



ii $A \cup (C \cap B')$



2 **a** $(A' \cup B') \cap C$ **b** $A \cap (B' \cup C')$
c $(A' \cap B') \cap C$ **d** $A' \cap (B \cap C')$
e $(A' \cap C) \cup B$ **f** $A \cap (C' \cup B)'$
g $A \cap (B \cup C)'$ **h** $(A \cap C) \cup (A \cup (B \cup C))'$
i $(A \cup B)' \cap C$ **j** $A' \cap (B \cup C)'$

3 **a** 1 **b** 3
c 4 **d** 2
e 7 **f** 6
g 5 **h** 8

4 **a** 1, 2 ,4 **b** 3, 6, 7
c 1, 4, 7 **d** 2, 5, 6
e 3, 4, 7 **f** 2, 6, 8
g 2, 3, 6 **h** 4, 7, 8

Exercise 8G

1 6 study Biology **only** (that is "Biology, but not Chemistry")

2 14 study **exactly** one science (that is "Biology or Chemistry, but not **both**")

3 21 study **at least** one science (that is "Biology or Chemistry, or **both**")

4 21 study one science (that is "Biology or Chemistry, or **both**")

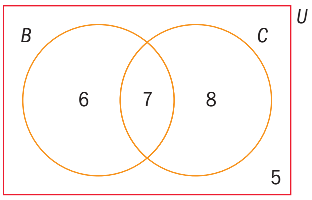
5 13 do not study Biology

6 11 do not study Chemistry

7 7 Chemists study Biology

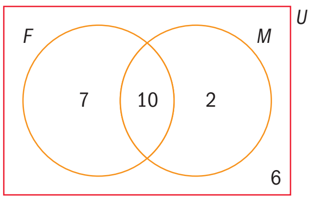
8 6 Biologists do not study Chemistry

9 14 science students do not study both Biology and Chemistry



Exercise 8H

1



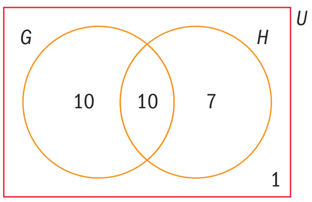
a 7 study French only.

b 19 study Malay or French or both.

c 6 study neither subject.

d 15 do not study both subjects.

2



a 28 are in the class.

b 11 do not study History.

c 10 study Geography but not History.

d 17 study Geography or History but not both.

3 Let x be the number of students who play both piano and violin.

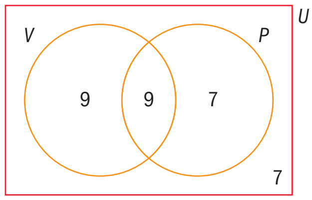
Then $18 - x$ play violin only

$16 - x$ play piano only

So $(18 - x) + x + (16 - x) + 7 = 32$

$41 - x = 32$

$x = 9$



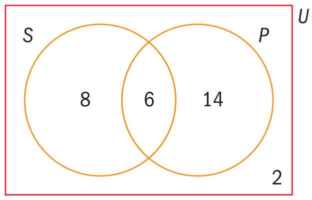
a 9 play the violin but not the piano.

b 14 do not play the violin.

c 7 play the piano but not the violin.

d 16 play the piano or the violin, but not both.

4



a Let x be the number of students who have studied both probability and set theory.
Then $20 - x$ studied probability only
$14 - x$ studied set theory only.

So $(20 - x) + x + (14 - x) + 2 = 30$

$36 - x = 30$

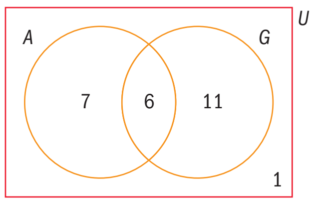
$x = 6$

6 have studied both topics.

b 22 have studied exactly one of these subjects.

c 8 have studied set theory, but not probability.

5



a Let x be the number of girls who have taken both aerobics and gymnastics.
Then $13 - x$ studied aerobics only
$17 - x$ studied gymnastics only

So $(13 - x) + (17 - x) + 1 = 25$

$31 - x = 25$

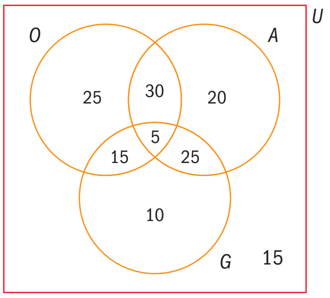
$x = 6$

6 have taken both activities.

b 11 have taken gymnastics but not aerobics.

c 24 have taken at least one of these activities.

Exercise 8I



1 **a** 70 liked exactly two of the three flavors of juice.

b 70 did not like orange juice.

c 55 liked one flavor of juice only.

d 25 did not like either orange or apple juice.

e 25 did not like orange juice and did not like apple juice.

f 75 liked at least two of the three flavors of juice.

g 70 liked fewer than two of the three flavours of juice.

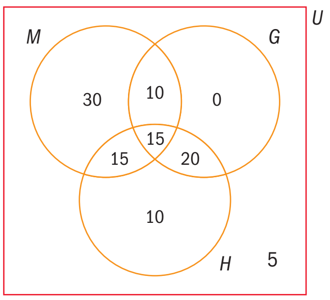
h 35

i 55

j 25

k 45

2



a 100 passed at least one subject.

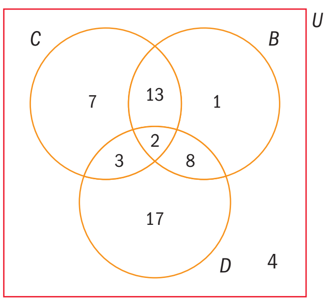
b 45 passed exactly 2 subjects.

c 20 passed geography and failed Mathematics.

d 15 passed all three subjects given that they passed two.

e 30 failed Mathematics given that they passed History.

3 4 are not fulfilling their responsibilities.



a 25 take part in one activity only.

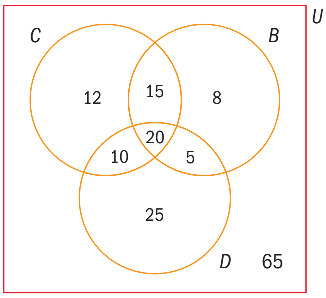
b 24 take part in exactly 2 activities.

c 29 do not take part in at least 2 activities.

d 5 take part in chess given that they take part in dominoes.

e 14 take part in backgammon given that they do not take part in dominoes.

4



a 50 ordered more than one type of rice.

b 65 did not order a rice dish from Fatty's Delight.

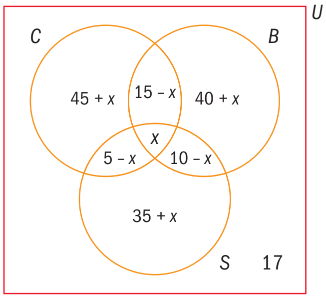
c 103 did not order chicken rice.

d 15 ordered duck rice and one other rice dish.

5 $65 + 40 + x + 10 - x + 35 + x + 17 = 170$

$167 + x = 170$

$x = 3$



a 129 take up one activity only.

b 24 take up at least two activities.

c 146 take part in fewer than two activities.

d 15 boulder given that they climb.

e 9 take up one other activity given that they swim.

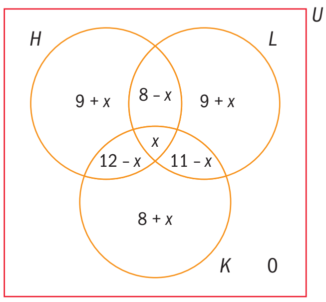
6 **a** Let x be the number that suffer from all 3 diseases.

Then $29 + 9 + x + 11 - x + 8 + x = 65$

$57 + x = 65$

$x = 8$

8 suffer from all three diseases.



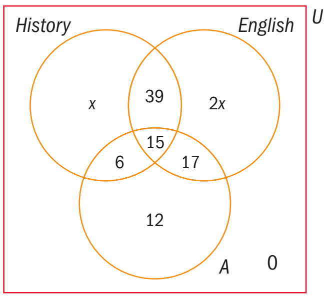
b 15 suffer from at least two diseases.

c 3 suffer from lung disease and exactly one other.

d 0 suffer from heart disease and lung disease but not kidney disease.

e 17 suffer from lung disease only.

7



a $3x + 89 = 116$

$3x = 27$

$x = 9$

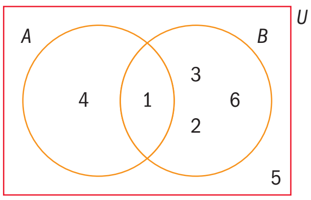
b $18 + 39 + 15 + 17 = 89$

Exercise 8J

1 **a** $A = \{1, 4\}$

b $B = \{1, 2, 3, 6\}$

c



d $P(A) = \frac{2}{6}$

e $P(B) = \frac{4}{6}$

f P(not a square number) $= \frac{4}{6}$

g P(both a square number and a factor of 6) $= \frac{1}{6}$

h P(either a square number or a factor of 6 or both) $= \frac{5}{6}$

i $P(A') = \frac{4}{6} = 1 - \frac{2}{6} = 1 - P(A)$

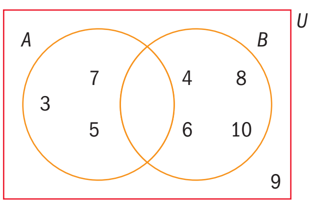
$P(A \cup B) = \frac{5}{6} = \frac{2}{6} + \frac{4}{6} - \frac{1}{6}$

$= P(A) + P(B) - P(A \cap B)$

2 **a** $A = \{3, 5, 7\}$

b $B = \{4, 6, 8, 10\}$

c



d $P(A) = \frac{3}{8}$

e $P(B) = \frac{4}{8}$

f $\frac{5}{8}$

g $\frac{4}{8}$

h 0

i $\frac{7}{8}$

j $P(A') = \frac{5}{8} = 1 - \frac{3}{8} = 1 - P(A)$

$P(B') = \frac{4}{8} = 1 - \frac{4}{8} = 1 - P(B)$

k $P(A \cup B) = \frac{7}{8} = \frac{3}{8} + \frac{4}{8} - \frac{0}{8}$

$= P(A) + P(B) - P(A \cap B)$

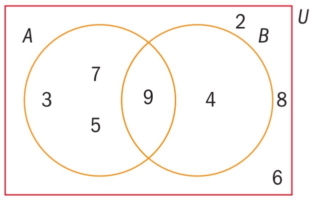
l $\frac{1}{8}$

m 1

n $P(A' \cup B') = 1 = \frac{5}{8} + \frac{4}{8} - \frac{1}{8}$

$= P(A') + P(B') - P(A' \cap B')$

3 **a** $A = \{3, 5, 7, 9\}$



b $B = \{4, 9\}$

d $P(A) = \frac{4}{8}$

e $P(B) = \frac{2}{8}$

f $\frac{1}{8}$

g $\frac{5}{8}$

h $P(A \cup B) = \frac{5}{8} = \frac{4}{8} + \frac{2}{8} - \frac{1}{8}$

$= P(A) + P(B) - P(A \cap B)$

4 A random experiment is: toss two unbiased coins.

a {HH, HT, TH, TT}

b $\frac{1}{4}, \frac{1}{2}, \frac{1}{4}$

5 A random experiment is: toss three unbiased coins.

a {HHH, HHT, HTH, THH, HTT, THT, TTH, TTT}

b $\frac{1}{8}, \frac{3}{8}, \frac{3}{8}, \frac{1}{8}$

6 A random experiment is: toss four unbiased coins.

a $\frac{1}{16}$

b $\frac{1}{16}$

c $\frac{4}{16}$

d $\frac{4}{16}$

e $1 - \left(\frac{1}{16} + \frac{4}{16} + \frac{4}{16} + \frac{1}{16}\right) = \frac{6}{16}$

f $\begin{Bmatrix} \text{HHHH, HHHT, HHTH, HTHH, HHTT,} \\ \text{HTHT, HTTH, HTTT, THHH, TTHH,} \\ \text{THTH, THHT, THTT, TTHT, TTTH,} \\ \text{TTTT} \end{Bmatrix}$

Exercise 8K

1 **a** $\frac{23}{40}$ **b** $\frac{5}{40}$ **c** $\frac{5}{40}$

d $\frac{15}{20}$ **e** $\frac{8}{23}$ **f** $\frac{8}{23}$

2 **a** $\frac{14}{30}$ **b** $\frac{8}{30}$ **c** $\frac{6}{10}$

d $\frac{8}{20}$ **e** $\frac{4}{16}$ **f** 0

3 **a** $\frac{8}{17}$ **b** $\frac{2}{17}$ **c** $\frac{8}{17}$

d $\frac{7}{9}$ **e** 0 **f** 1

4 **a** $\frac{12}{34}$ **b** $\frac{16}{34}$ **c** $\frac{28}{34}$

d $\frac{12}{22}$ **e** $\frac{6}{18}$ **f** $\frac{12}{22}$

5 **a** $\frac{13}{24}$ **b** $\frac{4}{24}$ **c** $\frac{8}{24}$

d $\frac{17}{24}$ **e** $\frac{7}{24}$ **f** $\frac{12}{24}$

g $\frac{9}{24}$

6 **a** $\frac{5}{22}$ **b** $\frac{18}{22}$ **c** $\frac{10}{15}$

d $\frac{3}{8}$

7 **a** $\frac{12}{28}$ **b** $\frac{4}{13}$ **c** $\frac{4}{16}$

d $\frac{3}{28}$ **e** $\frac{12}{21}$

8 **a** $\frac{12}{27}$ **b** $\frac{12}{20}$ **c** $\frac{7}{19}$

d $\frac{2}{7}$ **e** $\frac{12}{17}$

Exercise 8L

1 $A \cap B = \{1\}$ $\therefore$ NOT mutually exclusive

2 $A \cap B = \varnothing$ $\therefore$ mutually exclusive

3 $A \cap B = \{2\}$ $\therefore$ NOT mutually exclusive

4 $A \cap B = \varnothing$ $\therefore$ mutually exclusive

5 $A \cap B = \{9\}$ $\therefore$ NOT mutually exclusive

6 $A \cap B = \varnothing$ $\therefore$ mutually exclusive

7 $A \cap B = \{6\}$ $\therefore$ NOT mutually exclusive

8 $A \cap B = \varnothing$ $\therefore$ mutually exclusive

Exercise 8M

1 $P(A) = \frac{5}{9}$ $P(B) = \frac{3}{9} = \frac{1}{3}$ $P(A \cap B) = \frac{2}{9}$

$\frac{2}{9} \neq \frac{5}{9} \times \frac{1}{3}$ $\therefore$ not independent.

2 $P(A) = \frac{3}{6} = \frac{1}{2}$ $P(B) = \frac{2}{6} = \frac{1}{3}$ $P(A \cap B) = \frac{1}{6}$

$\frac{1}{6} = \frac{1}{2} \times \frac{1}{3}$ $\therefore$ independent.

3 $P(A) = \frac{4}{9}$ $P(B) = \frac{3}{9} = \frac{1}{3}$ $P(A \cap B) = \frac{1}{9}$

$\frac{1}{9} \neq \frac{4}{9} \times \frac{1}{3}$ $\therefore$ not independent.

4 $P(A) = \frac{6}{24} = \frac{1}{4}$ $P(B) = \frac{8}{24} = \frac{1}{3}$ $P(A \cap B) = \frac{2}{24} = \frac{1}{12}$

$\frac{1}{12} = \frac{1}{4} \times \frac{1}{3}$ $\therefore$ independent.

5 $P(C) = \frac{10}{18} = \frac{5}{9}$ $P(B) = \frac{11}{18}$ $P(C \cap B) = \frac{8}{18}$

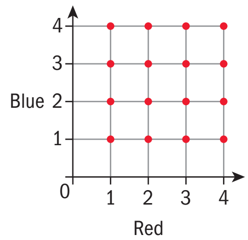
$\frac{8}{18} \neq \frac{5}{9} \times \frac{11}{18}$ $\therefore$ not independent.

6 $P(C) = \frac{20}{40} = \frac{1}{2}$ $P(P) = \frac{10}{40} = \frac{1}{4}$ $P(C \cap P) = \frac{8}{40} = \frac{1}{5}$

$\frac{1}{5} \neq \frac{1}{2} \times \frac{1}{4}$ $\therefore$ not independent.

Exercise 8N

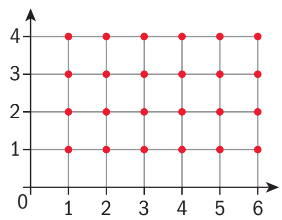
1



a $\frac{6}{16}$ or $\frac{3}{8}$ **b** $\frac{6}{16}$ or $\frac{3}{8}$

c $\frac{4}{16}$ or $\frac{1}{4}$ **d** $\frac{9}{16}$

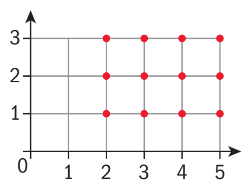
2



a $\frac{6}{24}$ **b** $\frac{13}{24}$ **c** $\frac{6}{24}$

d $\frac{11}{24}$ **e** $\frac{4}{24}$

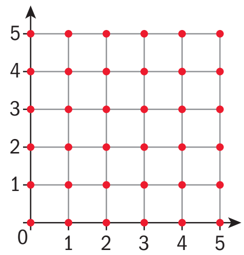
3



a $\frac{2}{12}$ **b** $\frac{4}{12}$ **c** $\frac{9}{12}$

d $\frac{5}{12}$ **e** $\frac{8}{12}$

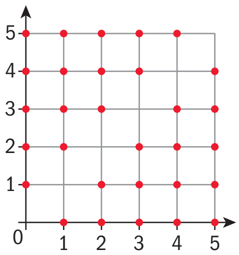
4



a $\frac{6}{36}$ **b** $\frac{23}{36}$ **c** $\frac{26}{36}$

d $\frac{13}{36}$ **e** $\frac{27}{36}$

5

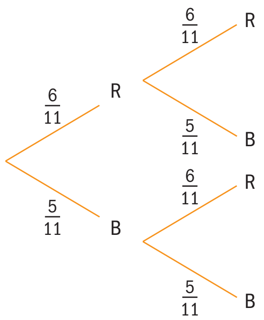


a 0 **b** $\frac{20}{30}$ **c** $\frac{22}{30}$

d $\frac{10}{30}$ **e** $\frac{24}{30}$

Exercise 80

1



a P(one red) $= \frac{6}{11}\times\frac{5}{11}+\frac{5}{11}\times\frac{6}{11}=\frac{60}{121}$

b P(at least one blue)

$= \frac{6}{11}\times\frac{5}{11}+\frac{5}{11}\times\frac{6}{11}+\frac{5}{11}\times\frac{5}{11}=\frac{85}{121}$

c $\frac{60}{121}$

d P(B second | one of each color)

$= \frac{P(\text{B second} \cap \text{one of each color})}{P(\text{one of each color})}$

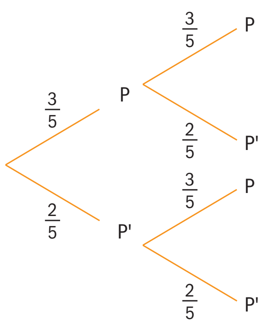
$= \frac{\frac{6}{11}\times\frac{5}{11}}{\frac{60}{121}} = \frac{1}{2}$

e P(B first | at least one blue)

$= \frac{P(\text{B first} \cap \text{at least one blue})}{P(\text{at least one blue})}$

$= \frac{\frac{5}{11}\times\frac{6}{11}+\frac{5}{11}\times\frac{5}{11}}{\frac{85}{121}} = \frac{55}{85} = \frac{11}{17}$

2



a P(exactly one prime)

$= \frac{3}{5}\times\frac{2}{5}+\frac{2}{5}\times\frac{3}{5}=\frac{12}{25}$

b P(at least one prime)

$= \frac{12}{25}+\frac{3}{5}\times\frac{3}{5}=\frac{21}{25}$

c P(two primes | at least one prime)

$= \frac{P(\text{two primes} \cap \text{at least one prime})}{P(\text{at least one prime})}$

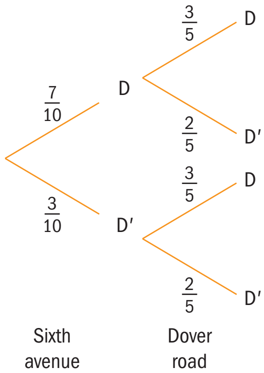
$= \frac{\frac{9}{25}}{\frac{21}{25}} = \frac{9}{21} = \frac{3}{7}$

d P(prime first | at least one prime)

$= \frac{P(\text{prime first and at least one prime})}{P(\text{at least one prime})}$

$= \frac{\left(\frac{3}{5}\times\frac{3}{5}+\frac{3}{5}\times\frac{2}{5}\right)}{\left(\frac{21}{25}\right)} = \frac{15}{21} = \frac{5}{7}$

3



a P(delayed once only)

$= \frac{7}{10}\times\frac{2}{5}+\frac{3}{10}\times\frac{3}{5}=\frac{23}{50}$

b P(no delay) $= \frac{3}{10}\times\frac{2}{5}=\frac{6}{50}=\frac{3}{25}$

c P(delayed at S.A | delayed exactly once)

$= \frac{P(\text{delayed at S.A.} \cap \text{delayed exactly once})}{P(\text{delayed exactly once})}$

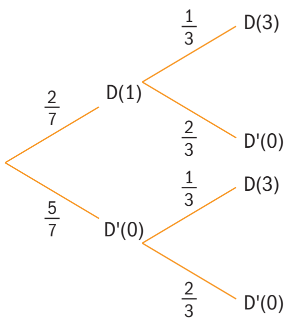
$= \frac{\frac{7}{10}\times\frac{2}{5}}{\frac{23}{50}} = \frac{14}{23}$

d P(delayed at S.A | delayed)

$= \frac{P(\text{delayed at S.A.} \cap \text{delayed})}{P(\text{delayed})}$

$= \frac{\frac{7}{10}\times\frac{3}{5}+\frac{7}{10}\times\frac{2}{5}}{1-\frac{6}{50}} = \frac{35}{44}$

4



a P(no delays) $= \frac{5}{7}\times\frac{2}{3}=\frac{10}{21}$

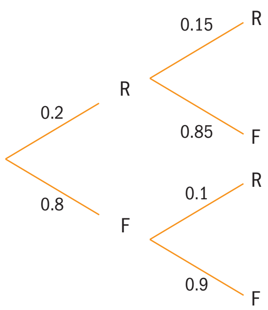
b P(one delay) $= \frac{2}{7}\times\frac{2}{3}+\frac{5}{7}\times\frac{1}{3}=\frac{9}{21}=\frac{3}{7}$

c P(delayed at A | delayed) $= \frac{P(\text{delayed at A} \cap \text{delayed})}{P(\text{delayed})}$

$= \frac{\frac{2}{7}\times\frac{1}{3}+\frac{2}{7}\times\frac{2}{3}}{1-\frac{10}{21}} = \frac{6}{11}$

d DD or D′D: $\frac{2}{7}\times\frac{1}{3}+\frac{5}{7}\times\frac{1}{3}=\frac{1}{3}$

5



a P(at least one fine day)

$= 1 - 0.2 \times 0.15 = 0.97$

b P(fine today | at least one fine day)

$= \dfrac{\text{P(fine today} \cap \text{at least one fine day)}}{\text{P(at least one fine day)}}$

$= \dfrac{0.8 \times 0.1 + 0.8 \times 0.9}{0.97} = \dfrac{80}{97}$

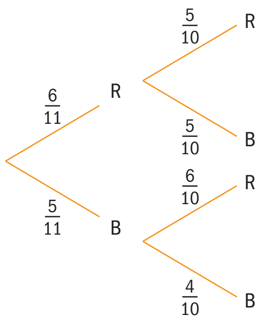
c P(both fine | at least one fine)

$= \dfrac{\text{P(both fine} \cap \text{at least one fine)}}{\text{P(at least one fine)}}$

$= \dfrac{0.8 \times 0.9}{0.97} = \dfrac{72}{97}$

Exercise 8P

1



a P(exactly one red)

$= \dfrac{6}{11} \times \dfrac{5}{10} + \dfrac{5}{11} \times \dfrac{6}{10} = \dfrac{60}{110}$

b P(at least one blue)

$= 1 - \dfrac{6}{11} \times \dfrac{5}{10} = \dfrac{80}{110}$

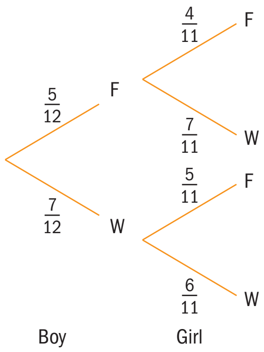
c P(one of each colour) $= \dfrac{60}{110}$

d P(blue second | one of each colour)

$= \dfrac{\frac{6}{11} \times \frac{5}{10}}{\frac{60}{110}} = \dfrac{1}{2}$

e P(blue first | at least one blue) $= \dfrac{\frac{5}{11}}{\frac{80}{110}} = \dfrac{5}{8}$

2



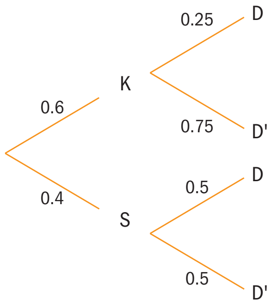
a P(2 faulty) $= \dfrac{5}{12} \times \dfrac{4}{11} = \dfrac{20}{132} = \dfrac{5}{33}$

b P(at least one faulty) $= 1 - \dfrac{7}{12} \times \dfrac{6}{11} = \dfrac{15}{22}$

c P(Girl F | exactly one F)

$= \dfrac{\frac{7}{12} \times \frac{5}{11}}{\frac{5}{12} \times \frac{7}{11} + \frac{7}{12} \times \frac{5}{11}} = \dfrac{35}{70} = \dfrac{1}{2}$

3



a P(delayed) $= 0.6 \times 0.25 + 0.4 \times 0.5$

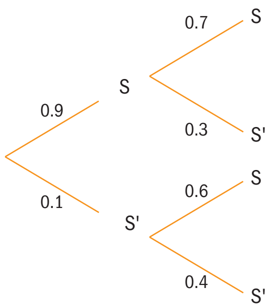
$= 0.35$

b $P(S \cap D') = 0.4 \times 0.5 = 0.2$

c $P(K \mid D) = \dfrac{P(K \cap D)}{P(D)} = \dfrac{0.6 \times 0.25}{0.35} = \dfrac{3}{7}$

d $P(S \mid D') = \dfrac{P(S \cap D')}{P(D')} = \dfrac{0.4 \times 0.5}{0.65} = \dfrac{20}{65} = \dfrac{4}{13}$

4



a $P(SS) = 0.9 \times 0.7 = 0.63$

b P(one S) $= 0.9 \times 0.3 + 0.1 \times 0.6$

$= 0.33$

c P(snow today | exactly one snow)

$= \dfrac{0.9 \times 0.3}{0.33} = \dfrac{27}{33} = \dfrac{9}{11}$

d P(snow today | at least one snow)

$= \dfrac{0.9}{1 - 0.1 \times 0.4} = \dfrac{90}{96} = \dfrac{15}{16}$

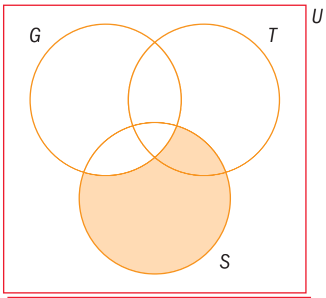
5 BR or RR $\dfrac{5}{8} \times \dfrac{3}{7} + \dfrac{3}{8} \times \dfrac{2}{7} = \dfrac{21}{56} = \dfrac{3}{8}$

Review exercise

Paper 1 style questions

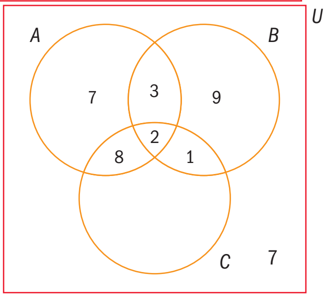
1 **a** **i** 6 **ii** 5 **iii** 10 **iv** 24

b



$G' \cap S$

2 **a, b**



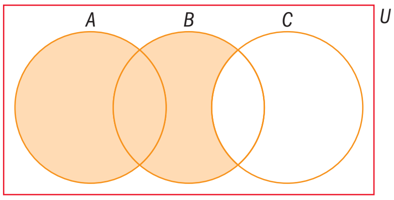
c 3

3 **a** F **b** T

c T **d** T

e T **f** T

4 **a**



$A \cup (B \cap C')$

b $4 + 6 + 5 = 15$

c **i** 5, 10, 15, 20

ii 10, 20, 30

5 For example:

a $-3, 4$

b $\frac{3}{4}, \pi$

c $\frac{2}{3}, \frac{-7}{10}$

d $\frac{1}{2}, \frac{5}{6}$

e $\pi, \sqrt{2}$

f $\pi, \sqrt{2}$

6 **a** $\frac{4}{60} = \frac{1}{15}$

b $1 - \frac{1}{15} = \frac{14}{15}$

c $\frac{16}{20} = \frac{4}{5}$

7 **a** $\frac{3}{15} = \frac{1}{15}$

b $\frac{3}{14}$

c $\frac{4}{15} \times \frac{3}{14} = \frac{2}{35}$

8 **a** 12

b $\frac{1}{2} \times \frac{1}{2} = \frac{1}{4}$

c $\frac{2}{6} = \frac{1}{3}$

9 **a** $3400 \le w < 3700$

b $\frac{5}{50} = \frac{1}{10}$

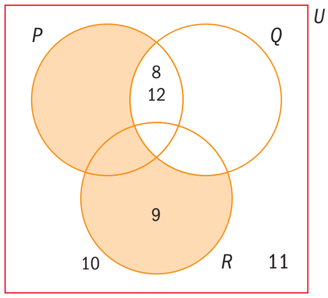
c $\frac{45}{50} = \frac{9}{10}$

d $\frac{20}{45} = \frac{4}{9}$

Paper 2 style questions

1 **a** $U = \{8, 9, 10, 11, 12\}$

b, e

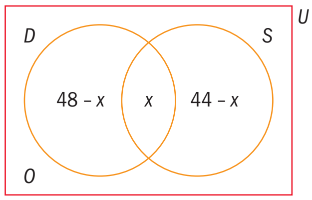


c **i** none

ii none

d numbers that are either multiples of 4 or factors of 24.

2 **a** **i**



ii $48 - x + x + 44 - x = 70$

$92 - x = 70$

$x = 22$

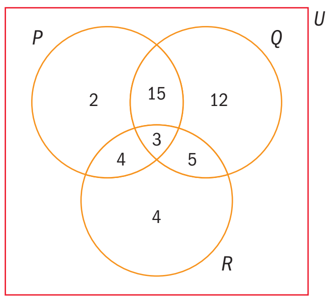
iii Members who did not attend both Drama and Sport

iv $P(D \text{ or } S) = \left[\frac{48-22}{70} + \frac{44-22}{70}\right] = \frac{48}{70} = \frac{24}{35}$

b **i** $\frac{30}{70} = \frac{3}{7}$

ii $\frac{12}{70} = \frac{6}{35}$

3 **a**



b $50 - 45 = 5$

c **i** $\frac{35}{50} = \frac{7}{10}$

ii $\frac{29}{50}$

iii $\frac{6}{24} = \frac{1}{4}$

d $\frac{3}{50} \times \frac{2}{49} = \frac{3}{1225}$

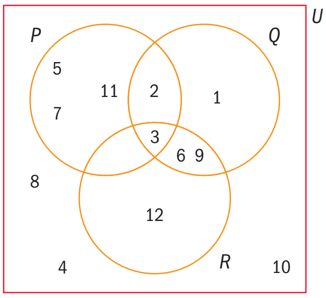
4 **a** **i** $P = \{2, 3, 5, 7, 11\}$

ii $Q = \{1, 2, 3, 6, 9\}$

iii $R = \{3, 6, 9, 12\}$

iv $P \cap Q \cap R = \{3\}$

b



c **i** $\{2, 3, 5, 6, 7, 9, 11\}$

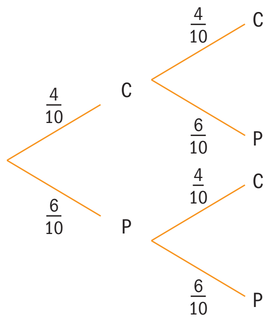
ii $\{1, 4, 8, 10\}$

iii $\{4, 8, 10\}$

d **i** $\frac{5}{12}$ **ii** $\frac{3}{12}$

iii $\frac{4}{12}$ **iv** $\frac{2}{5}$

5 **a**



b **i** $\frac{4}{10} \times \frac{4}{10} = \frac{16}{100}$

ii $\frac{4}{10} \times \frac{6}{10} + \frac{6}{10} \times \frac{4}{10} = \frac{48}{100}$

c **i** $a = 8 \quad b = 9$

ii 0

iii 1

d $\frac{1}{2} \times \frac{4}{10} + \frac{1}{2} \times \frac{1}{10} = \frac{5}{20} = \frac{1}{4}$

6 **a** **i** $\frac{13}{60}$

ii $\frac{16}{60}$

iii $\frac{42}{60}$

b **i** $\frac{4}{60} + \frac{7}{60} + \frac{16}{60} = \frac{27}{60}$

ii $\frac{9}{13}$

c **i** $\frac{36}{60} \times \frac{35}{59} = \frac{21}{59}$

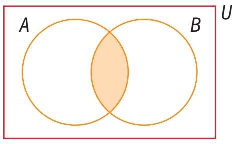
ii $\frac{45}{60} \times \frac{44}{59} = \frac{33}{59}$

9 Logic

Answers

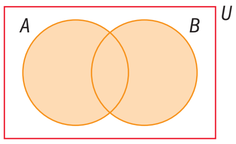
Skills check

1 **a**



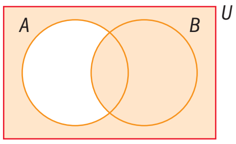
$A \cap B$

b



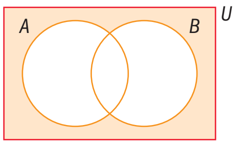
$A \cup B$

c



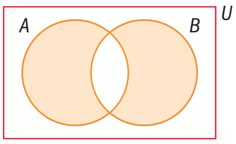
$A' \cup B$

d



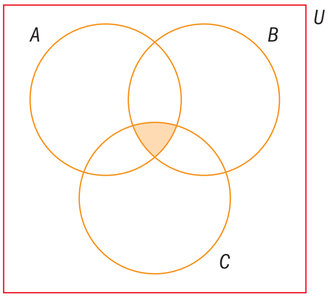
$(A \cup B)'$

e



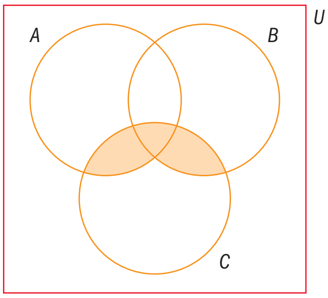
$(A \cap B') \cup (A' \cap B)$

2 **a**



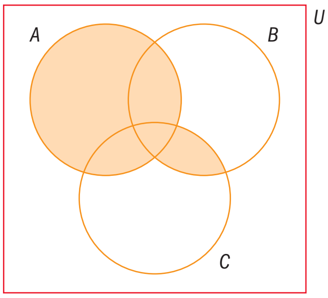
$A \cap B \cap C$

b



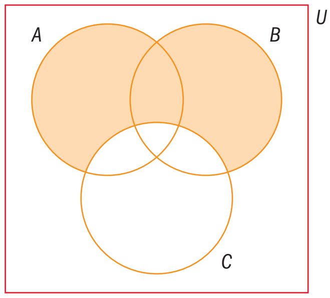
$(A \cup B) \cap C$

c



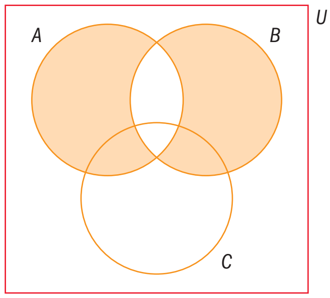
$A \cup (B \cap C)$

d



$(A \cup B) \cap C'$

e



$(A \cap B') \cup (A' \cap B)$

Exercise 9A

The following are statements:

1, 4, 5, 6, 7, 9, 10, 11, 12, 13, 14

Exercise 9B

1 exclusive
2 exclusive
3 inclusive
4 inclusive
5 inclusive
6 exclusive
7 inclusive
8 exclusive
9 exclusive
10 exclusive
11 exclusive
12 exclusive

Exercise 9C

1 **a** The student is not a council member.
b She does not own a mobile phone.
c n is not a prime number.
d ABCD is not a parallelogram.
e Surabaya is not the capital of Indonesia.

2 **a** This word starts with a consonant.
b There is an odd numbers of pages in this book
c This price is exclusive of sales tax
d This shape is a triangle or has more than 4 sides
e He walked at a variable speed.

3 **a** **i** No, the negation of p would be that Chihiro obtained any mark other than the highest.

ii No, the test could be not difficult without being easy.

iii No, Sahana could have scored 50% on the test.

iv Yes

v No, Nishad could have scored the average mark on the test.

b No

c Yes, Yes

4 **a** x is less than or equal to five

b y is greater than or equal to seven

c z is less than ten

d b is more than 19

5 **a** zero is neither positive nor negative.

b x is greater than or equal to zero.

6 **a** Courtney was present in school on Friday.

b This chair is not broken.

c The hockey team won or drew their match.

d The soccer team did not win the tournament.

e The hotel has running water.

7 **a** His signature is legible.

b James is either younger than or the same age as me.

c The class contains at least eight boys.

d Her family name begins with a letter other than P.

e He has fewer than two sisters.

8 **a** X is a male doctor

b X is female but she is not a doctor

c X is a married woman

d X is a single man

e R is a positive rotation of at most 90°

f R is a positive rotation of at most 90° or a negative rotation.

Exercise 9D

1 **a** Susan speaks French and Spanish

b Susan does not speak French and does speak Spanish.

c Susan speaks French and does not speak Spanish.

d Susan does not speak French and does not speak Spanish.

e Susan does not speak both French and Spanish.

2 **a** Jorge speaks Portuguese and Mei Ling speaks Malay.

b Jorge does not speak Portuguese and Mei Ling does speak Malay.

c Jorge speaks Portuguese and Mei Ling does not speak Malay.

d Jorge does not speak Portuguese and Mei Ling does not speak Malay.

e It is not true that both Jorge speaks Portuguese and Mei Ling speaks Malay.

3 **a** All dogs bark and all flowers are yellow.

b Not all dogs bark and all flowers are yellow.

c All dogs bark and not all flowers are yellow.

d Not all dogs bark and not all flowers are yellow.

e It is not true that both all dogs bark and all flowers are yellow.

4 **a** China is in Africa and Rwanda is in Asia.

b China is not in Africa and Rwanda is in Asia.

c China is in Africa and Rwanda is not in Asia.

d China is not in Africa and Rwanda is not in Asia.

e It is not true that both China is in Africa and Rwanda is in Asia.

5 **a** Chicago is the largest city in Canada and Jakarta is the largest city in Indonesia.

b Chicago is not the largest city in Canada and Jakarta is the largest city in Indonesia.

c Chicago is the largest city in Canada and Jakarta is not the largest city in Indonesia.

d Chicago is not the largest city in Canada and Jakarta is not the largest city in Indonesia.

e It is not true that both Chicago is the largest city in Canada and Jakarta is the largest city in Indonesia.

6 Yes, since x could equal 5.

7 **b** (all rectangles are parallelograms)

8 If ABC is right-angled at C then $AB^2 = AC^2 + BC^2$

a, b and **c** cannot be true, **d** and **e** must be true

9 **a** and **d** cannot be true, **e** must be true.

10

p	$\neg p$	$p \wedge \neg p$
T	F	F
F	T	F

11 $r = p \wedge \neg q$

p	q	¬q	r
T	T	F	F
T	F	T	T
F	T	F	F
F	F	T	F

12 $r = p \wedge q$

p	q	r	n
T	T	T	20
T	F	F	12
F	T	F	15
F	F	F	11

Exercise 9E

1 **a** **i** $x < 36$ or $x = 36$

ii $x < 36$ or $x = 36$ but not both

b **i**

2 **a** **i** $p \vee q$ **ii** $p \veebar q$ **iii** $q \vee r$

iv $(r \vee q) \wedge \neg p$

b No, since as 2 statements cannot both be true.

3 **a** **i** $p \vee q$ **ii** $p \veebar q$ **iii** $p \vee r$

iv $q \veebar r$ **v** $p \vee q \vee r$ **vi** $(p \vee q) \wedge \neg r$

b **i** 1, 2, 3, 4, 6, 9, 12, 18, 24, 30, 36

ii 1, 2, 3, 4, 9, 24, 30

iii 1, 4, 6, 9, 12, 16, 18, 24, 25, 30, 36

iv 2, 3, 6, 12, 16, 18, 25

v 1, 2, 3, 4, 6, 9, 12, 16, 18, 24, 25, 30, 36

vi 2, 3, 6, 12, 18, 24, 30

4 **a** $p \vee q$ **b** $r \veebar q$ **c** $p \vee r$ **d** $r \wedge q$

5 **a** $p \vee \neg q$ **b** $\neg p \wedge \neg q$

6 **a** x ends in zero or x is not divisible by 5 (eg.10, 13)

b x ends in zero or x is not divisible by 5 but not both (eg.10, 13)

c x ends in zero and is not divisible by 5 (necessarily false)

d x ends in zero and is divisible by 5 (eg.10)

e x does not ends in zero and is not divisible by 5 (eg.13)

7 **a** **i** $p \wedge q$ **ii** $p \veebar q$

iii $p \vee q$ **iv** $\neg p \vee \neg q$

v $\neg (p \vee q)$ **vi** $\neg (p \wedge q)$

vii $\neg p \wedge \neg q$

b **i** statement **i**

ii statement **iii**

iii statements **v** and **vii**

iv statements **iv** and **vi**

Exercise 9F

1 **a** **i** $p \wedge q$

p	q	p ∧ q
T	T	T
T	F	F
F	T	F
F	F	F

ii $p \veebar q$

p	q	p ⊻ q
T	T	F
T	F	T
F	T	T
F	F	F

iii $p \vee q$

p	q	p ∨ q
T	T	T
T	F	T
F	T	T
F	F	F

iv $\neg p \vee \neg q$

p	q	¬p	¬q	¬p ∨ ¬q
T	T	F	F	F
T	F	F	T	T
F	T	T	F	T
F	F	T	T	T

v $\neg (p \vee q)$

p	q	(p ∨ q)	¬(p ∨ q)
T	T	T	F
T	F	T	F
F	T	T	F
F	F	F	T

vi $\neg (p \wedge q)$

p	q	(p ∧ q)	¬(p ∧ q)
T	T	T	F
T	F	F	T
F	T	F	T
F	F	F	T

vii $\neg p \wedge \neg q$

p	q	¬p	¬q	¬p ∧ ¬q
T	T	F	F	F
T	F	F	T	F
F	T	T	F	F
F	F	T	T	T

b I am not studying both French and Chinese.

2 **a**

p	¬p	¬(¬p)
T	F	T
F	T	F

The 1st and 3rd columns are identical, therefore $\neg(\neg p) \Leftrightarrow p$

b

p	p	$(p \wedge p)$
T	T	T
F	F	F

The 1st and 3rd columns are identical, therefore $p \wedge p \Leftrightarrow p$

c

p	q	$(p \wedge q)$	$p \vee (p \wedge q)$
T	T	T	T
T	F	F	T
F	T	F	F
F	F	F	F

The 1st and 4th columns are identical, therefore $p \vee (p \wedge q) \Leftrightarrow p$

d

p	q	$p \vee q$	$\neg p$	$(\neg p \wedge q)$	$p \vee (\neg p \wedge q)$
T	T	T	F	F	T
T	F	T	F	F	T
F	T	T	T	T	T
F	F	F	T	F	F

The 3rd and 6th columns are identical, therefore $p \vee (\neg p \wedge q) \Leftrightarrow p \vee q$

3

p	q	$\neg p$	$\neg q$	$(p \wedge \neg q)$	$(\neg p \wedge q)$	$(p \wedge \neg q) \vee (\neg p \wedge q)$
T	T	F	F	F	F	F
T	F	F	T	T	F	T
F	T	T	F	F	T	T
F	F	T	T	F	F	F

$(p \wedge \neg q) \vee (\neg p \wedge q) \Leftrightarrow p \veebar q$

4 a

p	$\neg p$	$p \vee \neg p$
T	F	T
F	T	T

$p \vee \neg p$ is a tautology

b

p	$\neg p$	$p \wedge \neg p$
T	F	F
F	T	F

$p \wedge \neg p$ is a contradiction

c

p	p	$(p \wedge p)$	$p \wedge (p \wedge p)$
T	T	T	T
F	F	F	F

$p \wedge (p \wedge p)$ is neither a tautology nor a contradiction.

d

p	q	$(p \vee q)$	$\neg p$	$\neg q$	$(\neg p \wedge \neg q)$	$(p \vee q) \vee (\neg p \wedge \neg q)$
T	T	T	F	F	F	T
T	F	T	F	T	F	T
F	T	T	T	F	F	T
F	F	F	T	T	T	T

$(p \vee q) \vee (\neg p \wedge \neg q)$ is a tautology

e

p	q	$\neg p$	$\neg q$	$(p \vee \neg q)$	$(\neg p \wedge q)$	$(p \vee \neg q) \vee (\neg p \wedge q)$
T	T	F	F	T	F	T
T	F	F	T	T	F	T
F	T	T	F	F	T	T
F	F	T	T	T	F	T

$(p \vee \neg q) \vee (\neg p \wedge q)$ is a tautology

f

p	q	$\neg p$	$\neg q$	$(p \vee \neg q)$	$(\neg p \wedge \neg q)$	$(p \vee \neg q) \wedge (\neg p \wedge \neg q)$
T	T	F	F	T	F	F
T	F	F	T	T	F	F
F	T	T	F	F	F	F
F	F	T	T	T	T	T

$(p \vee \neg q) \wedge (\neg p \wedge \neg q)$ is neither a tautology nor a contradiction

g

p	q	$\neg p$	$(\neg p \vee q)$	$(p \wedge q)$	$(\neg p \vee q) \wedge (p \wedge q)$
T	T	F	T	T	T
T	F	F	F	F	F
F	T	T	T	F	F
F	F	T	T	F	F

$(\neg p \vee q) \wedge (p \wedge q)$ is neither a tautology nor a contradiction

h

p	q	$(p \wedge q)$	$\neg p$	$\neg q$	$(\neg p \wedge \neg q)$	$(p \wedge q) \wedge (\neg p \wedge \neg q)$
T	T	T	F	F	F	F
T	F	F	F	T	F	F
F	T	F	T	F	F	F
F	F	F	T	T	T	F

$(p \wedge q) \wedge (\neg p \wedge \neg q)$ is a contradiction

Exercise 9G

1

p	q	r	$(q \wedge r)$	$p \vee (q \wedge r)$
T	T	T	T	T
T	T	F	F	T
T	F	T	F	T
T	F	F	F	T
F	T	T	T	T
F	T	F	F	F
F	F	T	F	F
F	F	F	F	F

$p \vee (q \wedge r)$ is neither

2

p	q	r	$\neg q$	$(p \vee \neg q)$	$(p \vee \neg q) \vee r$
T	T	T	F	T	T
T	T	F	F	T	T
T	F	T	T	T	T
T	F	F	T	T	T
F	T	T	F	F	T
F	T	F	F	F	F
F	F	T	T	T	T
F	F	F	T	T	T

$(p \vee \neg q) \vee r$ is neither

3

p	q	r	$\neg r$	$p \wedge q$	$p \wedge \neg r$	$(p \wedge q) \vee (p \wedge \neg r)$
T	T	T	F	T	F	T
T	T	F	T	T	T	T
T	F	T	F	F	F	F
T	F	F	T	F	T	T
F	T	T	F	F	F	F
F	T	F	T	F	F	F
F	F	T	F	F	F	F
F	F	F	T	F	F	F

$(p \wedge q) \vee (p \wedge \neg r)$ is neither

4

p	q	r	$(p \vee q)$	$\neg q$	$(r \wedge \neg q)$	$(p \vee q) \vee (r \wedge \neg q)$
T	T	T	T	F	F	T
T	T	F	T	F	F	T
T	F	T	T	T	T	T
T	F	F	T	T	F	T
F	T	T	T	F	F	T
F	T	F	T	F	F	T
F	F	T	F	T	T	T
F	F	F	F	T	F	F

$(p \vee q) \vee (r \wedge \neg q)$ is neither

5

p	q	r	$(p \wedge r)$	$\neg r$	$(q \wedge \neg r)$	$(p \wedge r) \wedge (q \wedge \neg r)$
T	T	T	T	F	F	F
T	T	F	F	T	T	F
T	F	T	T	F	F	F
T	F	F	F	T	F	F
F	T	T	F	F	F	F
F	T	F	F	T	T	F
F	F	T	F	F	F	F
F	F	F	F	T	F	F

$(p \wedge r) \wedge (q \wedge \neg r)$ is a contradiction

6

p	q	r	$\neg p$	$(\neg p \vee q)$	$(p \wedge r)$	$(\neg p \vee q) \vee (p \wedge r)$
T	T	T	F	T	T	T
T	T	F	F	T	F	T
T	F	T	F	F	T	T
T	F	F	F	F	F	F
F	T	T	T	T	F	T
F	T	F	T	T	F	T
F	F	T	T	T	F	T
F	F	F	T	T	F	T

$(\neg p \vee q) \vee (p \wedge r)$ is neither

7

p	q	r	$\neg p$	$(\neg p \vee q)$	$(p \vee r)$	$(\neg p \vee q) \wedge (p \vee r)$
T	T	T	F	T	T	T
T	T	F	F	T	T	T
T	F	T	F	F	T	F
T	F	F	F	F	T	F
F	T	T	T	T	T	T
F	T	F	T	T	F	F
F	F	T	T	T	T	T
F	F	F	T	T	F	F

$(\neg p \vee q) \wedge (p \vee r)$ is neither

8

p	q	r	$(p \vee q)$	$(p \vee r)$	$(p \vee q) \wedge (p \vee r)$
T	T	T	T	T	T
T	T	F	T	T	T
T	F	T	T	T	T
T	F	F	T	T	T
F	T	T	T	T	T
F	T	F	T	F	F
F	F	T	F	T	F
F	F	F	F	F	F

$(p \vee q) \wedge (p \vee r)$ is neither. It is equivalent to $p \vee (q \wedge r)$

Exercise 9H

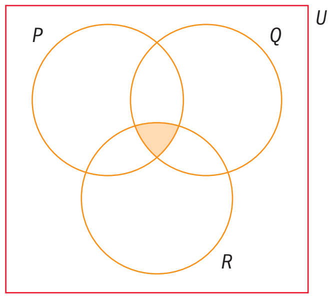
1

p	q	r	$(p \wedge q)$	$(p \wedge q) \wedge r$
T	T	T	T	T
T	T	F	T	F
T	F	T	F	F
T	F	F	F	F
F	T	T	F	F
F	T	F	F	F
F	F	T	F	F
F	F	F	F	F

p	q	r	$(q \wedge r)$	$p \wedge (q \wedge r)$
T	T	T	T	T
T	T	F	F	F
T	F	T	F	F
T	F	F	F	F
F	T	T	T	F
F	T	F	F	F
F	F	T	F	F
F	F	F	F	F

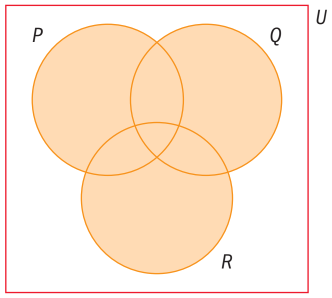
$(p \wedge q) \wedge r \Leftrightarrow p \wedge (q \wedge r)$ and brackets are not required.

2



$(P \cap Q) \cap R = P \cap (Q \cap R)$

3



$(P \cup Q) \cup R = P \cup (Q \cup R)$

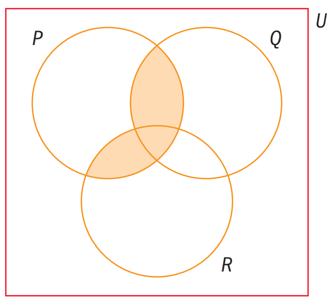
4

p	**q**	**r**	**(q ∨ r)**	**p ∨ (q ∨ r)**
T	T	T	T	T
T	T	F	T	T
T	F	T	T	T
T	F	F	F	F
F	T	T	T	F
F	T	F	T	F
F	F	T	T	F
F	F	F	F	F

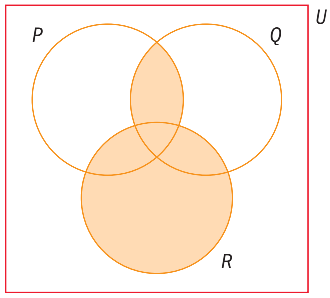
p	**q**	**r**	**(p ∧ q)**	**(p ∧ q) ∨ r**
T	T	T	T	T
T	T	F	T	T
T	F	T	F	T
T	F	F	F	F
F	T	T	F	T
F	T	F	F	F
F	F	T	F	T
F	F	F	F	F

The statements $p \vee (q \vee r)$ and $(p \wedge q) \vee r$ are not equivalent therefore brackets are required.

5



$P \cap (Q \cup R)$



$(P \cap Q) \cup R$

$P \cap (Q \cup R) \neq (P \cap Q) \cup R$

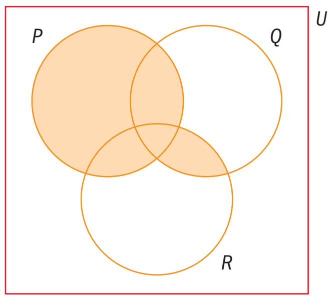
6

p	**q**	**r**	**(q ∧ r)**	**p ∨ (q ∧ r)**
T	T	T	T	T
T	T	F	F	T
T	F	T	F	T
T	F	F	F	T
F	T	T	T	T
F	T	F	F	F
F	F	T	F	F
F	F	F	F	F

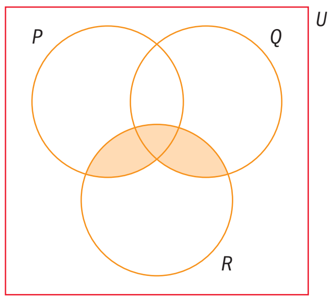
p	**q**	**r**	**(p ∨ q)**	**(p ∨ q) ∧ r**
T	T	T	T	T
T	T	F	T	F
T	F	T	T	T
T	F	F	T	F
F	T	T	T	T
F	T	F	T	F
F	F	T	F	F
F	F	F	F	F

The statements $p \vee (q \wedge r)$ and $(p \vee q) \wedge r$ are not equivalent therefore brackets are required.

7



$P \cup (Q \cap R)$



$(P \cup Q) \cap R$

$P \cup (Q \cap R) \neq (P \cup Q) \cap R$

8

p	q	r	$\neg p$	$\neg q$	$\neg r$	$(\neg p \wedge q)$	$(\neg q \wedge r)$	$(\neg r \wedge p)$	$(\neg p \wedge q) \vee (\neg q \wedge r) \vee (\neg r \wedge p)$
T	T	T	F	F	F	F	F	F	F
T	T	F	F	F	T	F	F	T	T
T	F	T	F	T	F	F	T	F	T
T	F	F	F	T	T	F	F	T	T
F	T	T	T	F	F	T	F	F	T
F	T	F	T	F	T	T	F	F	T
F	F	T	T	T	F	F	T	F	T
F	F	F	T	T	T	F	F	F	F

p	q	r	$\neg p$	$\neg q$	$\neg r$	$(\neg p \vee q)$	$(\neg q \vee r)$	$(\neg r \vee p)$	$(\neg p \vee q) \wedge (\neg q \vee r) \wedge (\neg r \vee p)$
T	T	T	F	F	F	T	T	T	T
T	T	F	F	F	T	T	F	T	F
T	F	T	F	T	F	F	T	T	F
T	F	F	F	T	T	F	T	T	F
F	T	T	T	F	F	T	T	F	F
F	T	F	T	F	T	T	F	T	F
F	F	T	T	T	F	T	T	F	F
F	F	F	T	T	T	T	T	T	T

The statements $(\neg p \wedge q) \vee (\neg q \wedge r) \vee (\neg r \wedge p)$ and $(\neg p \vee q) \wedge (\neg q \vee r) \wedge (\neg r \vee p)$ are not equivalent.

Exercise 9I

1

p	q	$p \wedge q$	$p \Rightarrow p \wedge q$
T	T	T	T
T	F	F	F
F	T	F	T
F	F	F	T

$p \Rightarrow p \wedge q$ is invalid

p	q	$p \vee q$	$p \Rightarrow p \vee q$
T	T	T	T
T	F	T	T
F	T	T	T
F	F	F	T

$p \Rightarrow p \vee q$ is a tautology

2

p	q	$p \wedge q$	$p \wedge q \Rightarrow p$
T	T	T	T
T	F	F	T
F	T	F	T
F	F	F	T

$p \wedge q \Rightarrow p$ is a tautology

p	q	$p \vee q$	$p \vee q \Rightarrow p$
T	T	T	T
T	F	T	T
F	T	T	F
F	F	F	T

$p \vee q \Rightarrow p$ is invalid

3

p	q	$p \vee q$	$p \wedge q$	$(p \vee q \Rightarrow p)$	$(p \Rightarrow p \wedge q)$	$(p \vee q \Rightarrow p) \wedge (p \Rightarrow p \wedge q)$
T	T	T	T	T	T	T
T	F	T	F	T	F	F
F	T	T	F	F	T	F
F	F	F	F	T	T	T

$(p \vee q \Rightarrow p) \wedge (p \Rightarrow p \wedge q)$ is invalid

4

p	q	$p \wedge q$	$(p \wedge q \Rightarrow p)$	$(p \Rightarrow p \wedge q)$	$(p \wedge q \Rightarrow p) \wedge (p \Rightarrow p \wedge q)$
T	T	T	T	T	T
T	F	F	T	F	F
F	T	F	T	T	T
F	F	F	T	T	T

$(p \wedge q \Rightarrow p) \wedge (p \Rightarrow p \wedge q)$ is invalid

5

p	q	$p \wedge q$	$(p \wedge q \Rightarrow p)$	$(p \Rightarrow p \wedge q)$	$(p \wedge q \Rightarrow p) \vee (p \Rightarrow p \wedge q)$
T	T	T	T	T	T
T	F	F	T	F	T
F	T	F	T	T	T
F	F	F	T	T	T

$(p \wedge q \Rightarrow p) \vee (p \Rightarrow p \wedge q)$ is a tautology

6

p	q	$p \wedge q$	$\neg(p \wedge q)$	$\neg p$	$\neg q$	$\neg p \vee \neg q$	$\neg(p \wedge q) \Rightarrow \neg p \vee \neg q$
T	T	T	F	F	F	F	T
T	F	F	T	F	T	T	T
F	T	F	T	T	F	T	T
F	F	F	T	T	T	T	T

$\neg(p \wedge q) \Rightarrow \neg p \vee \neg q$ is a tautology

7

p	q	$p \vee q$	$\neg(p \vee q)$	$\neg p$	$\neg q$	$\neg p \vee \neg q$	$\neg(p \vee q) \Rightarrow \neg p \vee \neg q$
T	T	T	F	F	F	F	T
T	F	T	F	F	T	T	T
F	T	T	F	T	F	T	T
F	F	F	T	T	T	T	T

$\neg(p \vee q) \Rightarrow \neg p \vee \neg q$ is a tautology

8

p	q	$\neg p$	$\neg q$	$\neg p \vee \neg q$	$p \wedge q$	$\neg(p \wedge q)$	$\neg p \vee \neg q \Rightarrow \neg(p \wedge q)$
T	T	F	F	F	T	F	T
T	F	F	T	T	F	T	T
F	T	T	F	T	F	T	T
F	F	T	T	T	F	T	T

$\neg p \vee \neg q \Rightarrow \neg(p \wedge q)$ is a tautology

9

p	q	$p \vee q$	$\neg(p \vee q)$	$\neg p$	$\neg q$	$\neg p \wedge \neg q$	$\neg(p \vee q) \Rightarrow \neg p \wedge \neg q$
T	T	T	F	F	F	F	T
T	F	T	F	F	T	F	T
F	T	T	F	T	F	F	T
F	F	F	T	T	T	T	T

$\neg(p \vee q) \Rightarrow \neg p \wedge \neg q$ is a tautology

Exercise 9J

1 p Madeline plugs the CD player in
q Madeline blows a fuse
The argument is $[(p \Rightarrow q) \wedge \neg p] \Rightarrow \neg q$

p	q	$p \Rightarrow q$	$\neg p$	$(p \Rightarrow q) \wedge \neg p$	$\neg q$	$[(p \Rightarrow q) \wedge \neg p] \Rightarrow \neg q$
T	T	T	F	F	F	T
T	F	F	F	F	T	T
F	T	T	T	T	F	F
F	F	T	T	T	T	T

The argument is invalid

2 p Muamar applies weed killer
q yield increases
The argument is $[(p \Rightarrow q) \wedge q] \Rightarrow p$

p	q	$p \Rightarrow q$	$(p \Rightarrow q) \wedge q$	$[(p \Rightarrow q) \wedge q] \Rightarrow p$
T	T	T	T	T
T	F	F	F	T
F	T	T	T	F
F	F	T	F	T

The argument is invalid

3 p Isaac passes the Maths test
q Isaac drops out of the IB diploma
The argument is $[(p \vee q) \wedge \neg q] \Rightarrow p$

p	q	$p \vee q$	$\neg q$	$(p \vee q) \wedge \neg q$	$[(p \vee q) \wedge \neg q] \Rightarrow p$
T	T	T	F	F	T
T	F	T	T	T	T
F	T	T	F	F	T
F	F	F	T	F	T

The argument is a tautology and is valid.

4 p you like music
p you go to tonight's concert
r you buy some CDs.
The argument is $[(p \Rightarrow q) \wedge (q \Rightarrow r) \wedge \neg r] \Rightarrow \neg p$

p	q	r	$p \Rightarrow q$	$q \Rightarrow r$	$\neg r$	$(p \Rightarrow q) \wedge (q \Rightarrow r) \wedge \neg r$	$\neg p$	$[(p \Rightarrow q) \wedge (q \Rightarrow r) \wedge \neg r] \Rightarrow \neg p$
T	T	T	T	T	F	F	F	T
T	T	F	T	F	T	F	F	T
T	F	T	F	T	F	F	F	T
T	F	F	F	T	T	F	F	T
F	T	T	T	T	F	F	T	T
F	T	F	T	F	T	F	T	T
F	F	T	T	T	F	F	T	T
F	F	F	T	T	T	T	T	T

The argument is a tautology and is valid.

5 p a person has an annual medical
q many illnesses can be detected early
r many lives can be saved
The argument is $[(p \Rightarrow q) \wedge (q \Rightarrow r)] \Rightarrow (\neg p \Rightarrow \neg r)$

p	q	r	$p \Rightarrow q$	$q \Rightarrow r$	$(p \Rightarrow q) \wedge (q \Rightarrow r)$	$\neg p$	$\neg r$	$\neg p \Rightarrow \neg r$	$[(p \Rightarrow q) \wedge (q \Rightarrow r)] \Rightarrow (\neg p \Rightarrow \neg r)$
T	T	T	T	T	T	F	F	T	T
T	T	F	T	F	F	F	T	T	T
T	F	T	F	T	F	F	F	T	T
T	F	F	F	T	F	F	T	T	T
F	T	T	T	T	T	T	F	F	F
F	T	F	T	F	F	T	T	T	T
F	F	T	T	T	T	T	F	F	F
F	F	F	T	T	T	T	T	T	T

The argument is invalid.

6 p you are involved in a car accident
q your insurance premiums go up
r you have to sell your car
The argument is $[(p \Rightarrow q) \wedge (q \Rightarrow r)] \Rightarrow (\neg p \Rightarrow \neg r)$
Truth table-see question 5
The argument is invalid.

7 p Doctor underwood gives difficult tests

q the students fail

r the students complain to Ms Smart

s Doctor underwood is dismissed

The argument is $[(p \Rightarrow q) \wedge (q \Rightarrow r) \wedge (r \Rightarrow s)] \Rightarrow (\neg s \Rightarrow \neg p)$

p	q	r	s	$p \Rightarrow q$	$q \Rightarrow r$	$r \Rightarrow s$	$(p \Rightarrow q) \wedge (q \Rightarrow r) \wedge (r \Rightarrow s)$	$\neg s$	$\neg p$	$\neg s \Rightarrow \neg p$	$[(p \Rightarrow q) \wedge (q \Rightarrow r)] \wedge (r \Rightarrow s) \Rightarrow (\neg s \Rightarrow \neg f)$
T	T	T	T	T	T	T	T	F	F	T	T
T	T	T	F	T	T	F	F	T	F	F	T
T	T	F	T	T	F	T	F	F	F	T	T
T	T	F	F	T	F	T	F	T	F	F	T
T	F	T	T	F	T	T	F	F	F	T	T
T	F	T	F	F	T	F	F	T	F	F	T
T	F	F	T	F	T	T	F	F	F	T	T
T	F	F	F	F	T	T	F	T	F	F	T
F	T	T	T	T	T	T	T	F	T	T	T
F	T	T	F	T	T	F	F	T	T	T	T
F	T	F	T	T	F	T	F	F	T	T	T
F	T	F	F	T	F	T	F	T	T	T	T
F	F	T	T	T	T	T	T	F	T	T	T
F	F	T	F	T	T	F	F	T	T	T	T
F	F	F	T	T	T	T	T	F	T	T	T
F	F	F	F	T	T	T	T	T	T	T	T

The argument is a tautology and is valid.

Exercise 9K

1

p	q	$p \wedge q$	$\neg(p \wedge q)$	$\neg p$	$\neg q$	$(\neg p \vee \neg q)$	$\neg(p \wedge q) \Leftrightarrow (\neg p \vee \neg q)$
T	T	T	F	F	F	F	T
T	F	F	T	F	T	T	T
F	T	F	T	T	F	T	T
F	F	F	T	T	T	T	T

$\neg(p \wedge q) \Leftrightarrow (\neg p \vee \neg q)$ is a tautology.

2

p	q	$p \vee q$	$\neg(p \vee q)$	$\neg p$	$\neg q$	$(\neg p \wedge \neg q)$	$\neg(p \vee q) \Leftrightarrow (\neg p \wedge \neg q)$
T	T	T	F	F	F	F	T
T	F	T	F	F	T	F	T
F	T	T	F	T	F	F	T
F	F	F	T	T	T	T	T

$\neg(p \vee q) \Leftrightarrow (\neg p \wedge \neg q)$ is a tautology.

3

p	q	$p \wedge q$	$(p \wedge q) \Leftrightarrow p$
T	T	T	T
T	F	F	F
F	T	F	T
F	F	F	T

p	q	$p \vee q$	$(p \vee q) \Leftrightarrow p$
T	T	T	T
T	F	T	T
F	T	T	F
F	F	F	T

The statements are not equivalent.

4

p	q	$\neg q$	$p \wedge \neg q$	$\neg(p \wedge \neg q)$	$\neg p$	$\neg p \vee q$
T	T	F	F	T	F	T
T	F	T	T	F	F	F
F	T	F	F	T	T	T
F	F	T	F	T	T	T

The columns for $\neg(p \wedge \neg q)$ and $\neg p \vee q$ are identical so the statements are equivalent.

5

p	q	$\neg q$	$p \vee \neg q$	$\neg(p \vee \neg q)$	$\neg p$	$\neg p \wedge q$
T	T	F	T	F	F	F
T	F	T	T	F	F	F
F	T	F	F	T	T	T
F	F	T	T	F	T	F

The columns for $\neg(p \vee \neg q)$ and $\neg p \wedge q$ are identical so the statements are equivalent.

6

p	q	$\neg q$	$(p \vee \neg q)$	$\neg p$	$(\neg p \wedge q)$	$(p \vee \neg q) \Leftrightarrow (\neg p \wedge q)$
T	T	F	T	F	F	F
T	F	T	T	F	F	F
F	T	F	F	T	T	F
F	F	T	T	T	F	F

The statement is a contradiction.

7

p	q	$(p \vee q)$	$\neg(p \vee q)$	$(p \wedge q)$	$\neg(p \vee q) \Leftrightarrow (\neg p \wedge q)$
T	T	T	F	T	F
T	F	T	F	F	T
F	T	T	F	F	T
F	F	F	T	F	F

The statement is neither a tautology nor a contradiction.

8

p	q	$\neg q$	$(p \wedge \neg q)$	$\neg p$	$\neg(p \vee q)$	$(p \wedge \neg q) \Leftrightarrow \neg(p \vee q)$
T	T	F	F	F	T	F
T	F	T	T	F	F	F
F	T	F	F	T	T	F
F	F	T	F	T	T	F

The statement is a contradiction.

Exercise 9L

1

p	q	p	$q \Rightarrow p$
T	T	T	T
T	F	T	T
F	T	F	F
F	F	F	T

2

p	q	$\neg p$	$\neg q$	$\neg p \Rightarrow \neg q$
T	T	F	F	T
T	F	F	T	T
F	T	T	F	F
F	F	T	T	T

3

p	q	$\neg q$	$\neg p$	$\neg q \Rightarrow \neg p$
T	T	F	F	T
T	F	T	F	F
F	T	F	T	T
F	F	T	T	T

Exercise 9M

a Valid

Converse: If ABCD is a quadrilateral then ABCD is a square. This is invalid. Counter example: a rectangle.

Inverse: If ABCD is not a square, then ABCD is not a quadrilateral. This is invalid. Counter example: a rectangle

Contrapositive: If ABCD is not a quadrilateral, then ABCD is not a square. This is valid.

b Valid

Converse: If ABCD is a parallelogram, then ABCD is a rectangle. Invalid. Counter example: a parallelogram without right angles.

Inverse: If ABCD is not a rectangle, then ABCD is not a parallelogram. This is invalid. Counter example: a parallelogram without right angles.

Contrapositive: If ABCD is not a parallelogram, then ABCD is not a rectangle. Valid.

c Valid

Converse: If an integer is divisible by 2 then it is divisible by 4. Invalid. Counter example: 6.

Inverse: If a integer is not divisible by 4 then it is not divisible by 2. Invalid. Counter example: 6.

Contrapositive: If an integer is not divisible by two then it is not divisible by 4. Valid.

d Invalid Counter example: 6

Converse: If an integer is odd then it is divisible by 3. Invalid. Counter example: 5.

Inverse: If an integer is not divisible by three then it is not odd. Invalid. Counter example: 7.

Contrapositive: If an integer is not odd then it is not divisible by 3. Invalid. Counter example: 6

e Valid

Converse: If an integer is even then it is divisible by 2. Valid.

Inverse: If an integer is not divisible by 2 then it is not even. Valid.

Contrapositive: If an integer is not even then it is not divisible by 2. Valid.

f Valid

Converse: If an integer is divisible by twelve then it is divisible by both four and by three. Valid.

Inverse: If a integer is not divisible by both four and by three then it is not divisible by twelve. Valid.

Contrapositive: If an integer is not divisible by twelve then it is not divisible by both four and by three. Valid.

g Invalid eg: 4

Converse: If an integer is divisible by eight then it is divisible by both four and by two. Valid.

Inverse: If an integer is not divisible by both four and by 2 then it is not divisible by eight. Valid.

Contrapositive: If an integer is not divisible by eight then it is not divisible by both four and by two. Invalid eg: 4.

h Invalid eg: 1 and 3

Converse: If two integers are both even then the sum of the two integers is even. Valid.

Inverse: If the sum of the two integers is not even then the two integers are not both even. Valid.

Contrapositive: If two integers are not both even then the sum of the two integers is not even. Invalid eg: 1 and 3.

i Invalid eg: 2 and 3

Converse: If two integers are both even then the product of the two integers is even. Valid.

Inverse: If the product of the two integers is not even then the two integers are not both even. Valid.

Contrapositive: If two integers are not both even then the product of the two integers is not even. Invalid eg: 2 and 3.

j Valid

Converse: If one integer is odd and one integer is even then the sum of the two integers is odd. Valid.

Inverse: If the sum of the two integers is not odd, then the integers are either both odd or both even. Valid.

Contrapositive: If two integers are either both even or both odd then the sum of the two integers is not odd. Valid.

k Valid

Converse: If two integers are both odd then the product of the two integers is odd. Valid.

Inverse: If the product of the two integers is not odd, then the two integers are not both odd. Valid.

Contrapositive: If two integers are not both odd then the product of the two integers is not odd. Valid.

l Valid

Converse: If $a^2 + b^2 = c^2$ then triangle ABC is right angled. Valid

Inverse: If triangle ABC is not right angled, then $a^2 + b^2 \neq c^2$. Valid.

Contrapositive: If $a^2 + b^2 \neq c^2$ then triangle ABC is not right angled. Valid

m **Direct argument:** If an integer is odd then its square is odd. Valid.

Converse: If the square of an integer is odd, then the integer is odd. Valid.

Inverse: If an integer is not odd then its square is not odd. Valid

Contrapositive: If the square of an integer is not odd then the integer is not odd. Valid.

n Valid

Converse: If triangle ABC has three equal sides then triangle ABC has three equal angles. Valid.

Inverse: If triangle ABC does not have three equal angles then triangle ABC does not have three equal sides. Valid.

Contrapositive: If triangle ABC does not have three equal sides then triangle ABC does not have three equal angles. Valid.

o Invalid eg: a rhombus

Converse: If quadrilateral ABCD has four equal angles then ABCD has four equal sides.

Invalid eg: a rectangle

Inverse: If quadrilateral ABCD does not have four equal sides, then ABCD does not have four equal angles. Invalid eq: a rectangle

Contrapositive: If quadrilateral ABCD does not have four equal angles then ABCD does not have four equal sides. Invalid eg: a rhombus

p Invalid. eq: $x = -5$

Converse: If $x = 5$, then $x^2 = 25$ valid

Inverse: If $x^2 \neq 25$, then $x \neq 5$ valid

Contrapositive: If $x \neq 5$, then $x^2 \neq 25$ Invalid eg: $x = -5$

q Valid

Converse: If $x = 3$, then $x^3 = 27$. Valid

Inverse: If $x^3 \neq 27$, then $x \neq 3$. Valid

Contrapositive: If $x \neq 3$, then $x^3 \neq 27$. Valid.

r Invalid. eq. $x < -5$

Converse: If $x > 5$, then $x^2 > 25$. Valid

Inverse: If $x^2 \leq 25$, then $x \leq 5$. Valid

Contrapositive: If $x \leq 5$, then $x^2 \leq 25$. Invalid eg: $x < -5$

s Valid

Converse: If $x < 3$, then $x^3 < 27$. Valid

Inverse: If $x^3 \geq 27$, then $x \geq 3$. Valid

Contrapositive: If $x \geq 3$, then $x^3 \geq 27$. Valid.

Review exercise

Paper 1 style questions

1 **a**

p	q	$p \vee q$	$\neg(p \vee q)$	$\neg p$	$\neg q$	$\neg p \vee \neg q$	$\neg(p \vee q) \Rightarrow \neg p \vee \neg q$
T	T	T	F	F	F	F	T
T	F	T	F	F	T	F	T
F	T	T	F	T	F	F	T
F	F	F	T	T	T	T	T

Since every entry in the root column is T $\neg(p \vee q) \Rightarrow \neg p \wedge \neg q$ is a valid argument.

b She does not dance well and she does not sing beautifully

2 **a** If the train leaves from gate 2, then it leaves today and not from gate 8.

b $\neg r \Leftrightarrow (p \vee q)$

3 **a**

p	q	$p \Rightarrow q$	$\neg p$	$\neg q$	$\neg q \vee p$	$\neg p \vee q$
T	T	T	F	F	T	T
T	F	F	F	T	T	F
F	T	T	T	F	F	T
F	F	T	T	T	T	T

b $(p \Rightarrow q) \Leftrightarrow (\neg p \vee q)$

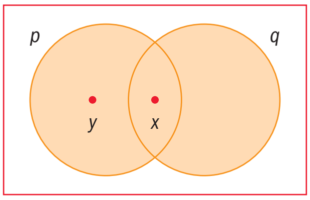
4 a

p	q	$\neg p$	$\neg p \vee q$
T	T	F	T
T	F	F	F
F	T	T	T
F	F	T	T

b i If $x > 3$ and $x^2 \not> 9$, p is T and q is F. From the table $\neg p \vee q$ is F.

ii If $x \not> 3$ and $x^2 > 9$, p is F and q is T. From the table $\neg p \vee q$ is T.

5 a, b



c i $\neg q \Rightarrow \neg p$ **ii** $\neg p \vee q$

iii $\neg q \Rightarrow p$ **iv** $p \wedge \neg q$

d Proposition **i** since it is the contrapositive of the given statement

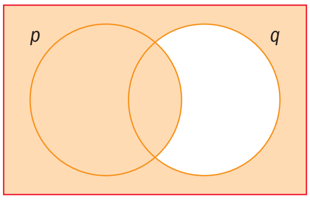
6 a i Picasso painted picture A or van Gogh did not paint picture A.

ii Picasso did not paint picture A and van Gogh painted picture A.

b

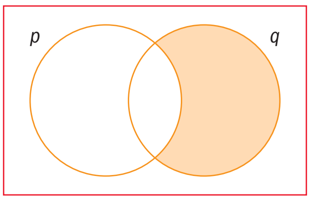
p	q	$\neg p$	$\neg q$	$p \vee \neg q$	$\neg p \wedge q$
T	T	F	F	T	F
T	F	F	T	T	F
F	T	T	F	F	T
F	F	T	T	T	F

c



p ∨ ¬q

$p \vee \neg q$



¬p ∧ q

$\neg p \wedge q$

d i

$(p \vee \neg q)$	$(\neg p \wedge q)$	$(p \vee \neg q) \Leftrightarrow (\neg p \wedge q)$
T	F	F
T	F	F
F	T	F
T	F	F

ii Using the Venn diagrams the regions representing $p \vee \neg q$ and $\neg p \wedge q$ do not overlap hence the truth values of $(p \vee \neg q) \Leftrightarrow (\neg p \wedge q)$ are all false.

e A logical contradiction.

7 a x is a multiple of 3 or a factor of 90 and is not a multiple of 5

b $r \Rightarrow (p \vee \neg q)$

c

p	q	r	$q \vee r$	$\neg p$	$(q \vee r) \wedge \neg p$
T	T	T	T	F	F
T	T	F	T	F	F
T	F	T	T	F	F
T	F	F	F	F	F
F	T	T	T	T	T
F	T	F	T	T	T
F	F	T	T	T	T
F	F	F	F	T	F

p	q	r	$\neg q$	$p \vee \neg q$	$r \Rightarrow (p \vee \neg q)$
T	T	T	F	T	T
T	T	F	F	T	T
T	F	T	T	T	T
T	F	F	T	T	T
F	T	T	F	F	F
F	T	F	F	F	T
F	F	T	T	T	T
F	F	F	T	T	T

d

p	q	r	x
F	T	T	3
F	T	F	12
F	F	T	2

e

p	q	r	$(q \vee r) \wedge \neg p$	$r \Rightarrow (p \vee \neg q)$
T	T	T	F	T
T	T	F	F	T
T	F	T	F	T
T	F	F	F	T
F	T	T	T	F
F	T	F	T	T
F	F	T	T	T
F	F	F	F	T

The statements are equivalent only in the cases

p	q	r
F	T	F
F	F	T

i.e x is not a multiple of 5 and is either a multiple of 3 or a factor of 90 (but not both).

10 Geometry and trigonometry 2

Answers

Skills check

1 **a** $\sin 20° = \frac{2}{x}$

$x = \frac{2}{\sin 20°}$

$x = 5.85 \text{ m } (3 \text{ s.f.})$

b $\tan y = \frac{7}{5.6}$

$y = \tan^{-1}\left(\frac{7}{5.6}\right)$

$y = 51.3° \ (3 \text{ s.f.})$

2 **a** use the sine or the cosine rule.

$\frac{120}{\sin 100°} = \frac{95}{\sin x}$

$\sin x = \frac{95 \sin 100°}{120}$

$x = \sin^{-1}\left(\frac{95 \sin 100°}{120}\right)$

$x = 51.2° (3 \text{ s.f.})$

b $180° - 51.22...° - 100° = 28.77...°$

$A = \frac{1}{2} \times 95 \times 120 \times \sin 28.77...$

$A = 2740 \text{ m}^2 \ (3 \text{ s.f.})$

3 to convert between units of area multiply or divide by 10^2 to convert between units of volume multiply or divide by 10^3

a $2.46 \text{ cm}^2 = 2.46 \times 10^2 \text{ mm}^2 = 246 \text{ mm}^2$

b $32\,000 \text{ m}^3 = 32000 \times 10^{-3} \text{dam}^3 = 32 \text{ dam}^3$

c $13.08 \text{ km}^2 = 13.08 \times 10^6 \text{ m}^2 = 13\,080\,000 \text{ m}^2$

d $0.0230 \text{ m}^3 = 0.0230 \times 10^6 \text{cm}^3 = 23000 \text{ cm}^3$

Exercise 10A

1 **a** **i** triangular prism;

ii 5 faces; 9 edges; 6 vertices;

iii 5 plane faces.

b **i** rectangular- based pyramid;

ii 5 faces; 8 edges; 5 vertices;

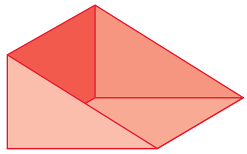
iii 5 plane faces.

c **i** hemisphere;

ii 2 faces; 1 edge; no vertex;

iii 1 plane face, 1 curve face.

2 **a**



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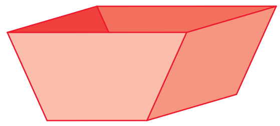

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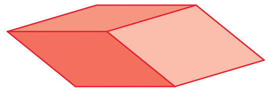
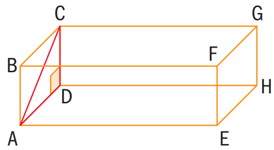


Exercise 10B

1 Mark first the three vertices of the triangle.

a



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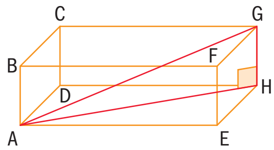

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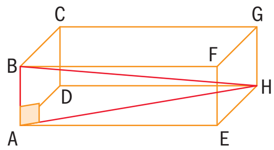
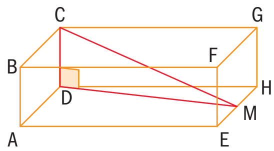
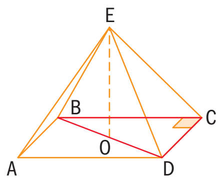


d



2 Mark first the three vertices of the triangle.

a



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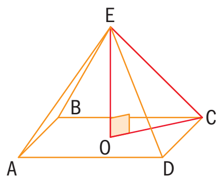

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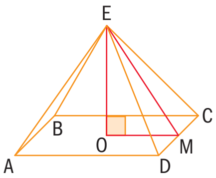


3 **a** DB is the hypotenuse of triangle ABD.

$DB^2 = AB^2 + AD^2$

$DB^2 = 4^2 + 6^2$

$DB = \sqrt{52}$ cm or 7.21 cm (3 s.f.)

b ED is the hypotenuse of triangle ADE.

$ED^2 = AD^2 + AE^2$

$ED^2 = 4^2 + 9^2$

$ED = \sqrt{97}$ cm or 9.85 cm (3 s.f.)

c DG is the hypotenuse of triangle DCG.

$DG^2 = DC^2 + CG^2$

$DG^2 = 6^2 + 9^2$

$DG = \sqrt{117}$ cm or 10.8 cm (3 s.f.)

d DF is the hypotenuse of triangle DBF.

$DF^2 = DB^2 + BF^2$

$DF^2 = \left(\sqrt{52}\right)^2 + 9^2$

$DF = \sqrt{133}$ cm or 11.5 cm (3 s.f.)

4 **a** AC is the hypotenuse of triangle ABC

$AC^2 = AB^2 + BC^2$

$AC^2 = (0.6)^2 + (0.6)^2$

$AC = \sqrt{0.72}$

$AC = 0.849$ m (3 s.f.)

b EOD is a right-angled triangle and OD is half of DB.

$ED^2 = EO^2 + OD^2$

$ED^2 = EO^2 + \left(\frac{DB}{2}\right)^2$

$ED^2 = (1.5)^2 + \left(\frac{\sqrt{0.72}}{2}\right)^2$

$ED = 1.56$ m (3 s.f.)

c EOM is a right-angled triangle.

$EM^2 = EO^2 + OM^2$

$EM^2 = (1.5)^2 + \left(\frac{0.6}{2}\right)^2$

$EM = 1.53$ m (3 s.f.)

5 Let A be any point on the circumference of the base. VOA is a right-angled triangle.

$VA^2 = AO^2 + OV^2$

$9^2 = 4^2 + OV^2$

$OV^2 = 9^2 - 4^2$

$OV^2 = 65$

$OV = \sqrt{65}$ cm or 8.06 cm (3 s.f.)

6 **a** ABC is a right-angled triangle.

$AC^2 = AB^2 + BC^2$

$AC^2 = (0.9)^2 + (0.7)^2$

$AC = 1.14$ m (3 s.f.)

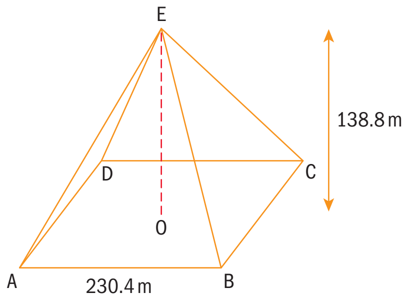
b The length of the longest fitness bar that can fit in the cupboard is the length of AG (or HB or CE or DF).

$AG^2 = AC^2 + CG^2$

$AG^2 = (1.1401\ldots)^2 + (1.5)^2$

$AG = 1.88$ m (3 s.f.)

7



a The base is a square.

$AC^2 = AB^2 + BC^2$

$AC^2 = (230.4)^2 + (230.4)^2$

$AC = 326$ m (3 s.f.)

b let M be the midpoint of BC

$EM^2 = EO^2 + OM^2$

$EM^2 = (138.8)^2 + \left(\frac{230.4}{2}\right)^2$

$EM^2 = 180$ m (3 s.f.)

c On the diagram EB is an inclined edge and EOB is a right-angled triangle.

$EB^2 = EO^2 + OB^2$

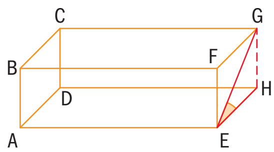
$EB^2 = EO^2 + \left(\frac{DB}{2}\right)^2$

$EB^2 = (138.8)^2 + 163^2$

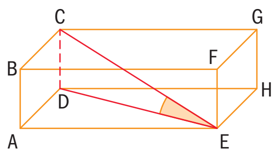
$EB = 214$ m (3 s.f.)

Exercise 10C

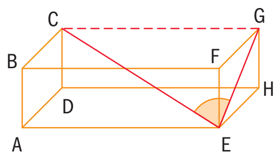
1 **a** Identify the plane and the line. Their point of intersection will be the vertex of the angle.



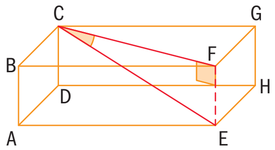
b



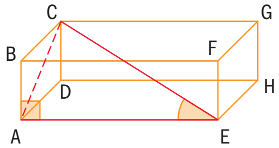
c



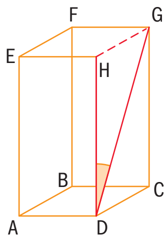
d Identify both lines. Their point of intersection will be the vertex of the angle.



e identify both lines. Their point of intersection will be the vertex of the angle.



2 a



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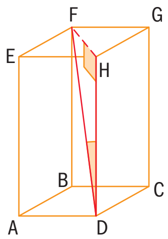

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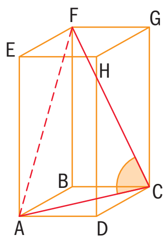

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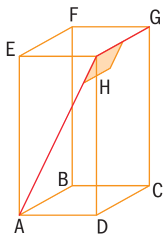
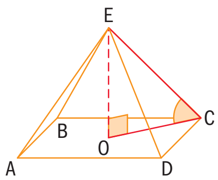
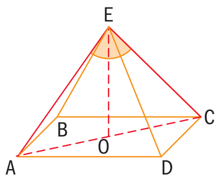


It is a 90° angle.

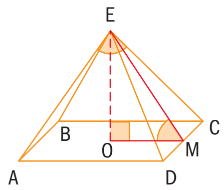
3 a Mark clearly the base and EC.



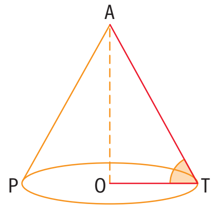
b Mark clearly the edges EC and AE.



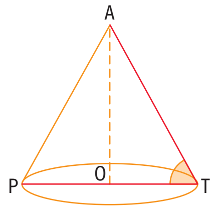
c



4 a AO is perpendicular to the base.

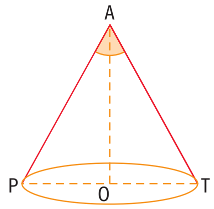


b



They are the same angle.

c



PAT is an isosceles triangle.

Exercise 10D

1 a i ABC is a right-angled triangle.

$AC^2 = AB^2 + BC^2$

$AC^2 = 4^2 + 10^2$

$AC^2 = 116$

$AC = \sqrt{116}$ cm or 10.8 cm

ii the angle is GAC

$\tan GAC = \frac{GC}{CA}$

$\tan GAC = \frac{3}{\sqrt{116}}$

$GAC = \tan^{-1}\left(\frac{3}{\sqrt{116}}\right)$

$GAC = 15.6°$ (3 s.f.)

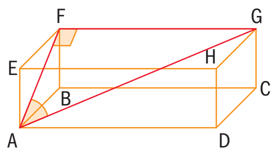
b i ADH is a right-angled triangle.

$AH^2 = AB^2 + BF^2$

$AH^2 = 4^2 + 3^2$

$AH = 5$ cm

ii The required angle is in one of the angles of triangle AHG.



$\tan FAG = \frac{FG}{AF}$

$\tan HAG = \frac{10}{5}$

$HAG = \tan^{-1}\left(\frac{10}{5}\right)$

$HAG = 63.4°$ (3 s.f.)

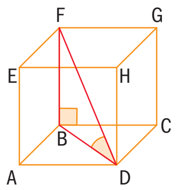
2 **a** ABCD is a square and BD is its diagonal.

$BD^2 = 2^2 + 2^2$

$BD^2 = 8$

$BD = \sqrt{8}$ m or 2.83 m (3 s.f.)

b The required angle is in one of the angles of triangle BHD.



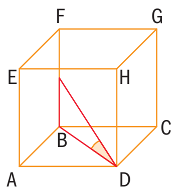
$\tan HDB = \frac{FB}{BD}$

$\tan HDB = \frac{2}{\sqrt{8}}$

$HDB = \tan^{-1}\left(\frac{2}{\sqrt{8}}\right)$

$HDB = 35.3°$ (3 s.f.)

c



$\tan MDB = \frac{MD}{BD}$

$\tan MDB = \frac{1}{\sqrt{8}}$

$MDB = \tan^{-1}\left(\frac{1}{\sqrt{8}}\right)$

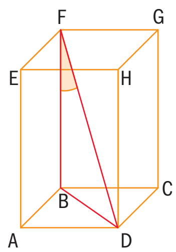
$MDB = 19.5°$ (3 s.f.)

3 **a** **i** $BD^2 = AB^2 + AD^2$

$BD^2 = 6^2 + 4^2$

$BD = \sqrt{52}$ cm or 7.21 cm (3 s.f.)

ii



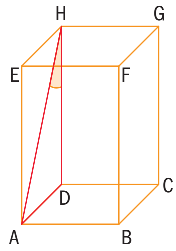
$\tan BDH = \frac{BD}{DH}$

$\tan BDH = \frac{\sqrt{52}}{9}$

$BDH = \tan^{-1}\left(\frac{\sqrt{52}}{9}\right)$

$BDH = 38.7°$ (3 s.f.)

b



$\tan AHD = \frac{AD}{DH}$

$\tan AHD = \frac{6}{9}$

$AHD = \tan^{-1}\left(\frac{6}{9}\right)$

$AHD = 33.7°$ (3 s.f.)

c **i** AH is the diagonal of ADHE.

$AH^2 = AD^2 + DH^2$

$AH^2 = 6^2 + 9^2$

$AH = \sqrt{117}$ cm or 10.8 cm (3 s.f.)

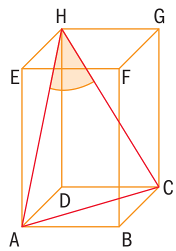
ii HC is the diagonal of CDHG.

$HC^2 = HD^2 + DC^2$

$HC^2 = 9^2 + 4^2$

$HC = \sqrt{97}$ cm or 9.85 cm (3 s.f.)

iii



$\cos AHC = \frac{AH^2 + HC^2 - AC^2}{2 \times AH \times HC}$

$\cos AHC = \frac{(\sqrt{117})^2 + (\sqrt{97})^2 - (\sqrt{52})^2}{2 \times \sqrt{117} \times \sqrt{97}}$

$\cos AHC = 0.76033...$

$AHC = \cos^{-1}(0.76033...)$

$AHC = 40.5°$ (3 s.f.)

4 **a** AC is the diagonal of ABCD.

$AC^2 = AB^2 + BC^2$

$AC^2 = 4^2 + 3^2$

$AC = 5$ cm

b AE is the hypotenuse of triangle AEO.

$AE^2 = AO^2 + OE^2$

$AE^2 = \left(\frac{AC}{2}\right)^2 + OE^2$

$AE^2 = \left(\frac{5}{2}\right)^2 + 7^2$

$AE = \sqrt{55.25}$

$AE = 7.43$ cm (3 s.f.)

c AEC is an isosceles triangle.

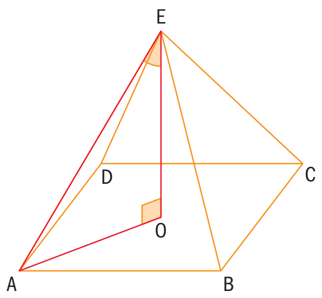
$\cos AEC = \dfrac{\left(\sqrt{55.25}\right)^2 + \left(\sqrt{55.25}\right)^2 - 5^2}{2\times\sqrt{55.25}\times\sqrt{55.25}}$

$\cos AEC = 0.77375...$

$AEC = \cos^{-1}(0.77375...)$

$AEC = 39.3°$ (3 s.f.)

d



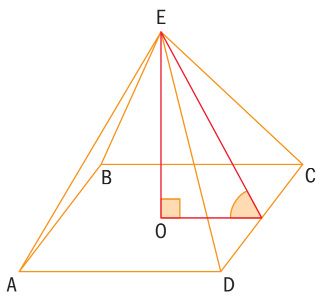
$\tan OAE = \frac{EO}{OA}$

$\tan OAE = \frac{7}{2.5}$

$OAE = \tan^{-1}\left(\frac{7}{2.5}\right)$

$OAE = 70.3°$ (3 s.f.)

e



$\tan EMO = \frac{EO}{OM}$

$\tan EMO = \frac{7}{2}$

$EMO = \tan^{-1}\left(\frac{7}{2}\right)$

$EMO = 74.1°$ (3 s.f.)

5 **a** ATO is a right-angled triangle.

$AT^2 = AO^2 + OT^2$

$AT^2 = 5^2 + 3^2$

$AT = \sqrt{34}$ cm or 5.83 cm (3 s.f.)

b $\tan ATO = \frac{AO}{OT}$

$\tan ATO = \frac{5}{3}$

$ATO = \tan^{-1}\left(\frac{5}{3}\right)$

$ATO = 59.0°$ (3 s.f.)

c PAT is an isosceles triangle and ∠ATO = ∠ATP

$PAT + 2\times ATO = 180°$

$PAT + 2\times(59.036...)° = 180°$

$PAT = 61.9°$ (3 s.f.)

6 **a** The base is a circle.

$A = \pi r^2$

$5 = \pi r^2$

$r = \sqrt{\frac{5}{\pi}}$ m or 1.26 m (3 s.f.)

b ∠PAT = 2 × ∠OAT.

$\tan OAT = \frac{TO}{OA}$

$\tan OAT = \frac{1.2615...}{2}$

$OAT = \tan^{-1}\left(\frac{1.2615...}{2}\right)$

$OAT = (32.243...)°$

$PAT = 2\times(32.243...)°$

$PAT = 64.5°$ (3 s.f.)

Exercise 10E

1 **a** All the faces are congruent.

Surface area = $6\times(2\times2)$

Surface area = 24 cm^2

b Surface area = $2\times(2.5\times1.5) + 2\times(2.5\times2) + 2\times(1.5\times2)$

Surface area = 23.5 m^2

c The three rectangles are congruent.

Surface area = 2 × Area of triangle ABC + 3 × Area of rectangle BEFC

Surface area = $2\times\left(\frac{1}{2}\times4\times4\times\sin60°\right) + 3\times(4\times5)$

Surface area = 73.9 cm^2

2 **a** Area of ABC $= \frac{1}{2}\times3\times3\times\sin120°$

Area of ABC = 3.90 cm^2 (3 s.f.)

b $AB^2 = AC^2 + BC^2 - 2\times AC\times BC\times\cos ACB$

$AB^2 = 3^2 + 3^2 - 2\times3\times3\times\cos120°$

$AB = \sqrt{27}$ or 5.20 cm (3 s.f.)

c Surface area = 2 × Area of triangle ABC + 2 × Area of rectangle ACFD + Area of rectangle ABED

Surface area = $2\times 3.8971... + 2\times(4\times3) + 4\times\sqrt{27}$

Surface area = 52.6 cm^2

3 **a** EOM is a right-angled triangle.

$EM^2 = EO^2 + OM^2$

$EM^2 = 6^2 + (2.5)^2$

$EM = 6.5$ cm

b EM is the height of triangle BCE.

Area of triangle BCE $= \frac{1}{2}\times5\times6.5$

Area of triangle BCE $= (16.25)\,cm^2$

c Surface area $= 4\times16.25 + 5^2$

Surface area = 90 cm^2

4 The cube has 6 congruent faces.

Let x be the side length of the cube.

$6x^2 = 600$

$x^2 = \frac{600}{6}$

$x^2 = 100$

$x = 10$

5 **a** Surface area $= 6\times(5.4)^2$

Surface area $= 174.96$ m^2 or 175 m^2 (3 s.f.)

b $175 = 1.75\times10^2$ m^2

6 **a** Surface area to be painted $= 2\times(3\times2.5)+4\times2.5+3\times4+4\times2.5-(2\times1.3+1\times1)$

Surface area to be painted $= 43.4$ m^2

b Amount of paint $= 1.2\times43.4$

Amount of paint = 52.08 litres = 53 litres when rounded up

c Cost in paint $= 4.60\times53$

Cost in paint = USD 243.80 (2 d.p.)

Exercise 10G

1 **a** Volume of cuboid $= l\times w\times h$

Volume of cuboid $= 12\times1.3\times1.5$

Volume of cuboid $= 23.4$ dm^3

b Volume of cuboid $= l\times w\times h$

Volume of cuboid $= 15\times3\times2$

Volume of cuboid $= 90$ m^3

c Volume of cube $= l^3$

Volume of cube $= 20^3$

Volume of cube $= 8000$ cm^3

d Volume of prism = area of cross section × height

Volume of prism $= \left(\frac{1}{2}\times8\times8\times\sin30^\circ\right)\times10$

Volume of prism $= 160$ cm^3

e $AB^2 + AC^2 = CB^2$

$AB^2 + 3^2 = 5^2$

$AB^2 = 5^2 - 3^2$

$AB = 4$ cm

Volume of prism = area of cross section × height

Volume of prism $= \frac{1}{2}\times(3\times4)\times2$

Volume of prism $= 12$ m^3

f Volume of prism = area of cross section × height

Volume of prism $= \frac{1}{2}\times(5\times7)\times12$

Volume of prism $= 210$ cm^3

2 **a** $\tan ACB = \frac{AB}{AC}$

$\tan 40^\circ = \frac{AB}{6}$

$AB = 6\tan40^\circ$

$AB = 5.03$ m (3 s.f.)

b Area of triangle ABC $= \frac{1}{2}\times(5.03\times6)$

Area of triangle ABC $= 15.1$ m^2 (3 s.f.)

c Volume of prism = area of cross section × height

Volume of prism $= 15.1\times10$

Volume of prism $= 151$ m^3 (3 s.f.)

3 **a** $COB = \frac{360^\circ}{6}$

$COB = 60^\circ$

b COB is an equilateral triangle.

CO = OB = BC = 5 cm

Area of triangle COB $= \frac{1}{2}\times5\times5\times\sin60^\circ$

Area of triangle COB $= 10.8$ cm^2 (3 s.f.)

c Area of regular hexagon $= 6\times10.825...$

Area of regular hexagon $= 65.0$ cm^2 (3 s.f.)

d Volume of prism = area of cross section × height

Volume of prism $= 65.0\times13.5$

Volume of prism $= 877$ cm^3 (3 s.f.)

4 **a** Volume of cuboid $= l\times w\times h$

Volume of cuboid $= 2x\times x\times 0.5x$

Volume of cuboid $= 2\times0.5\times x\times x\times x$

Volume of cuboid $= x^3$

b Volume of cuboid $= l\times w\times h$

Volume of cuboid $= x\times x\times 3x$

Volume of cuboid $= 3x^3$

c find first the area of the cross section.

area of cross section $= \frac{1}{2}\left(x\cdot\frac{3}{2}x\right)$

area of cross section $= \frac{1}{2}\left(\frac{3}{2}x^2\right)$

area of cross section $= \frac{3}{4}x^2$

Volume of prism = area of cross section × height

Volume of prism $= \left(\frac{3}{4}x^2\right)\times\frac{x}{2}$

Volume of prism $= \frac{3}{8}x^3$ or equivalent

d the cross section is a trapezium.

area of cross section $= (B + b)\,\frac{h}{2}$

area of cross section $= (3x + 2x)\,\frac{4}{2}$

area of cross section $= 10x$

Volume of prism = area of cross section × height

Volume of prism $= (10x) \times x$

Volume of prism $= 10x^2$

5 **a** Volume of cuboid $= l \times w \times h$

Volume of cuboid $= x \times x \times 25$

Volume of cuboid $= 25x^2$

b Volume of cuboid $= 25x^2$

Volume of cuboid $= 11025$

Therefore $25x^2 = 11025$

c $25x^2 = 11025$

$x^2 = \frac{11025}{25}$

$x^2 = 441$

$x = 21$

6 **a** Let x be the side length of the box.

$x^3 = 9261$

$x = \sqrt[3]{9261}$

$x = 21$

Therefore the side length is 21 cm.

b Surface area $= (5 \times 21^2)$ cm^2

Surface area $= 2205$ cm^2

Exercise 10H

1 **a** Volume of cylinder $= \pi r^2 h$

Volume of cylinder $= \pi \times 34^2 \times 65$

Volume of cylinder $= (75140\pi)$ mm^3 or 236000 mm^3 (3 s.f.)

b $r = \frac{1}{2}$ m

Volume of sphere $= \frac{4}{3}\pi r^3$

Volume of sphere $= \frac{4}{3}\pi \left(\frac{1}{2}\right)^3$

Volume of sphere $= \left(\frac{1}{6}\pi\right)$ m^3 or 0.524 m^3 (3 s.f.)

c Volume of cone $= \frac{1}{3}\pi r^2 h$

Volume of cone $= \frac{1}{3}\pi \times 2.5^2 \times 5$

Volume of cone $= \left(\frac{125}{12}\pi\right)$ m^3 or 32.7 m^3 (3 s.f.)

d Volume of cone $= \frac{1}{3}\pi r^2 h$

Volume of cone $= \frac{1}{3}\pi \times 6^2 \times 30$

Volume of cone $= 1130$ cm^2 (3 s.f.)

e Volume of hemisphere $= \frac{\text{volume of sphere}}{2}$

Volume of hemisphere $= \frac{\frac{4}{3}\pi r^3}{2}$

Volume of hemisphere $= \frac{\frac{4}{3}\pi \times 2.5^3}{2}$

Volume of hemisphere $= 32.7$ cm^3 (3 s.f.)

f this is a rectangular based pyramid.

Volume of pyramid $= \frac{1}{3}$ (Area of base × vertical height)

Volume of pyramid $= \frac{1}{3}(2 \times 3 \times 4)$

Volume of pyramid $= 8$ dm^3

2 **a** Volume of cylinder $= \pi r^2 h$

Volume of cylinder $= \pi \times 1.20^2 \times 3$

Volume of cylinder $= 13.6$ m^3 (3 s.f.)

b 13.6 m^3 = 13.6 × 1000 dm^3 = 13600 dm^3

1 litre = 1 dm^3

c 13600 dm^3 = 13600 litres

3 **a** Volume of pyrmid $= \frac{1}{3}$ (Area of base × vertical height)

Volume of pyramid $= \frac{1}{3}(x^2 \times h)$

Volume of pyramid $= \frac{x^2 \times h}{3}$ or equivalent

b Volume of cylinder $= \pi r^2 h$

Volume of cylinder $= \pi \times x^2 \times 2x$

Volume of cylinder $= 2\pi \times x^3$

c $r = \frac{6x}{2} = 3x$

Volume of cylinder $= \pi r^2 h$

Volume of cylinder $= \pi (3x)^2 \times x$

Volume of cylinder $= 9\pi x^3$

d $r = \frac{3x}{2}$

Volume of sphere $= \frac{4}{3}\pi r^3$

Volume of sphere $= \frac{4}{3}\pi \left(\frac{3x}{2}\right)^3$

Volume of sphere $= 4.5\pi x^3$

4 **a** Volume of pyramid $= \frac{1}{3}(\text{area of base} \times h)$

$84 = \frac{1}{3}(\text{area of base} \times 7)$

area of base $= \frac{84 \times 3}{7}$

area of base $= 36$ cm^2

b Area of AOB $= \frac{\text{Area of base}}{6}$

Area of AOB $= \frac{36}{6}$

Area of AOB $= 6$ cm^2

c $AOB = \frac{360^\circ}{6}$

$AOB = 60^\circ$

d Let x be the length of AB.

Area of AOB = 6 cm^2

Area of AOB = $\frac{1}{2} \times x \times x \times \sin 60^\circ$

Therefore

$6 = \frac{1}{2} \times x \times x \times \sin 60^\circ$

$6 = \frac{1}{2} x^2 \times \sin 60^\circ$

$6 = \frac{1}{2} x^2 \times \sin 60^\circ$

$x = 3.72$

AB = 3.72 cm (3 s.f.)

5 a Volume of sphere = $\frac{4}{3}\pi r^3$

$200 = \frac{4}{3}\pi r^3$

$\frac{200 \times \frac{3}{4}}{\pi} = r^3$

$r = \sqrt[3]{\frac{200 \times \frac{3}{4}}{\pi}}$

$r = 3.63$ cm (3 s.f.)

b $r = 3.63$ cm $= 3.63 \times 10$ mm $= 36.3$ mm

36.3 mm = 36 mm correct to the nearest millimetre.

6 a Volume of cylinder $= \pi r^2 h$

Volume of cylinder $= \pi \times 15^2 \times 30$

Volume of cylinder $= (6750)\pi$ cm^3 or 21200 cm^3 (3 s.f.)

b Volume of cuboid $= l \times w \times h$

Volume of cuboid $= 60 \times 20 \times 17$

Volume of cuboid = 20400 cm^3

There is not enough space as 21200 >20400.

Review exercise

Paper 1 style questions

1 a Surface area of ABCDEFGH = $2 \times (20 \times 42) + 2 \times (20 \times 34) + 2 \times (34 \times 42)$

Surface area of ABCDEFGH = 5896 cm^2

b Volume of cuboid $= l \times w \times h$

Volume of cuboid $= 34 \times 42 \times 20$

Volume of cuboid = 28560 cm^3

28560 cm^3 = 28560×10^{-3} dm^3 = 28.56 dm^3

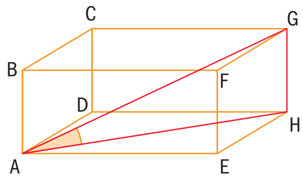
2 a AH is the hypotenuse of a triangle.

$AH^2 = AE^2 + EH^2$

$AH^2 = 10^2 + 4^2$

$AH = \sqrt{116}$ cm or 10.8 cm (3 s.f.)

b



$\tan HAG = \frac{5}{\sqrt{116}}$

$HAG = \tan^{-1}\left(\frac{5}{\sqrt{116}}\right)$

$HAG = 24.9^\circ$ (3 s.f.)

3 a AC is the diagonal of the base.

$AC^2 = AB^2 + BC^2$

$AC^2 = 4^2 + 5^2$

$AC = \sqrt{41}$ cm or 6.40 cm (3 s.f.)

b $EC^2 = EO^2 + OC^2$

$EC^2 = EO^2 + \left(\frac{AC}{2}\right)^2$

$EC^2 = 8^2 + \left(\frac{\sqrt{41}}{2}\right)^2$

$EC = \sqrt{74.25}$ cm or 8.62 cm (3 s.f.)

c AEC is an isosceles triangle.

$\cos AEC = \frac{AE^2 + EC^2 - AC^2}{2 \times AE \times EC}$

$\cos AEC = \frac{(\sqrt{74.25})^2 + (\sqrt{74.25})^2 - (\sqrt{41})^2}{2 \times \sqrt{74.25} \times \sqrt{74.25}}$

$AEC = \cos^{-1}\left(\frac{(\sqrt{74.25})^2 + (\sqrt{74.25})^2 - (\sqrt{41})^2}{2 \times \sqrt{74.25} \times \sqrt{74.25}}\right)$

$AEC = 43.6^\circ$ (3 s.f.)

4 a Let the midpoint be M

$EO^2 + OM^2 = EM^2$

$9^2 + 3^2 = EC^2$

$EC = \sqrt{90}$ cm or 9.49 cm (3 s.f.)

b Area of triangle BCE $= \frac{1}{2} \times 6 \times \sqrt{90}$

Area of triangle BCE = 28.5 cm^2 (3 s.f.)

c Surface area of pyramid

= 4 × area of triangle BEC + Area of base

Surface area of pyramid = 4 × 28.46....+ 6^2

Surface area of pyramid = 150 cm^2 (3 s.f.)

5 **a** Let x be the edge length of the cube.

Volume of cube = x^3

$512 = x^3$

$\sqrt[3]{512} = x$

$x = 8$ cm

b AC is the diagonal of the base.

$AC^2 = AB^2 + BC^2$

$AC^2 = 8^2 + 8^2$

$AC^2 = 128$

$AC = \sqrt{128}$ cm or 11.3 cm (3 s.f.)

c $AG^2 = AC^2 + CG^2$

$AG^2 = \left(\sqrt{128}\right)^2 + 8^2$

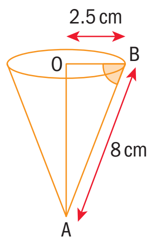
$AG^2 = 192$

$AG = \sqrt{192}$ cm or 13.9 cm (3 s.f.)

13.5 < 13.9, therefore the pencil fits in the cube.

6 Triangle AOB is a right-angled triangle.

a



$\cos OBA = \frac{2.5}{8}$

$OBA = \cos^{-1}\left(\frac{2.5}{8}\right)$

$OBA = 71.8°$ (3 s.f.)

b **i** $AB^2 = AO^2 + OB^2$

$8^2 = AO^2 + 2.5^2$

$AO^2 = 8^2 - 2.5^2$

$AO = \sqrt{57.75}$ cm or 7.60 cm (3 s.f.)

ii Volume of cone $= \frac{1}{3}\pi r^2 h$

Volume of cone $= \frac{1}{3}\pi \times 2.5^2 \times \sqrt{57.75}$

Volume of cone = 49.7 cm³ (3 s.f.)

7 **a** Area of triangle ABC $= \frac{1}{2} \times 2.4 \times 2.4 \times \sin 110°$

Area of triangle ABC = 2.71 m² (3 s.f.)

b Volume = area of cross section × height

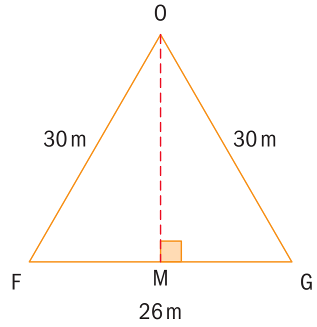
Volume = 2.706... × 3.5

Volume = 9.47m³ (3 s.f.)

Review exercise

Paper 2 style questions

1 **a**



triangle FGO is isosceles and OM is its height.

$OM^2 + MG^2 = GO^2$

$OM^2 + 13^2 = 30^2$

$OM^2 = 30^2 - 13^2$

$OM = \sqrt{731}$ m or 27.0 m (3 s.f.)

b The height of the tower is the addition of the height of the pyramid and the height of the cuboid.

Let P be the midpoint of the base of the pyramid.

$OP^2 + PM^2 = OM^2$

$OP^2 + 13^2 = \left(\sqrt{731}\right)^2$

$OP^2 = \left(\sqrt{731}\right)^2 - 13^2$

$OP^2 = 562$

$OP = \sqrt{562}$

Height of the tower = OP + height of cuboid

Height of the tower = $\sqrt{562}$ + 70

Height of the tower = 93.7 m (3 s.f.)

c $\cos OMP = \frac{13}{\sqrt{731}}$

$OMP = \cos^{-1}\left(\frac{13}{\sqrt{731}}\right)$

$OMP = 61.3°$ (3 s.f.)

d Surface area = 4 × (26 × 70) + 4 × $\left(\frac{1}{2} \times 26 \times \sqrt{731}\right)$

Surface area = 8685.9246...m²

Cost of cleaning = 78 × 8685.9246... m²

Cost of cleaning = USD 677502 (correct to the nearest dollar)

2 **a** Volume of hemisphere $= \dfrac{\frac{4}{3}\pi r^3}{2}$

Volume of hemisphere $= \dfrac{\frac{4}{3}\pi \times 3^3}{2}$

Volume of hemisphere $= \dfrac{\frac{4}{3}\times 3^3}{2}\pi$

Volume of hemisphere = (18π) cm^2

b Volume of cone $= \frac{1}{3}\times\pi\times 3^2\times h$

Volume of hemisphere = (18π) cm^3

Therefore

$\frac{2}{3}\left(\frac{1}{3}\times\pi\times 3^2\times h\right) = 18\pi$

$\frac{1}{3}\times\pi\times 3^2\times h = \dfrac{18\pi}{\frac{2}{3}}$

$\pi\times 3\times h = 27\pi$

$h = \dfrac{27\pi}{\pi\times 3}$

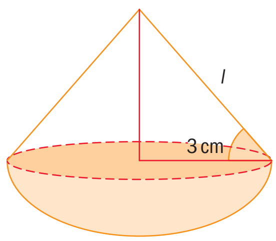
$h = 9$ cm

c $l^2 = 3^2 + 9^2$

$l^2 = 90$

$l = \sqrt{90}$ cm or 9.49 cm (3 s.f.)

d



Let the angle be α.

$\tan\alpha = \frac{9}{3}$

$\alpha = \tan^{-1}\left(\frac{9}{3}\right)$

$\alpha = 71.6°$ (3 s.f.)

e Volume of sculpture = Volume of hemisphere + Volume of cone

Volume of sculpture $= 18\pi + \frac{1}{3}\times\pi\times 3^2\times 9$

Volume of sculpture $= 18\pi + 27\pi$

Volume of sculpture = (45π) cm^3

Weight of sculpture $= 45\pi\times 10.8$

Weight of sculpture = 1530 grams (3 s.f.)

Therefore

1530 grams = 1.53 kg

3 **a** Volume of pyramid $= \frac{1}{3}$(area of base × height)

Volume of pyramid $= \frac{1}{3}(5^2\times 7)$

Volume of pyramid $= \frac{175}{3}$ cm^3 or 58.3 cm^3 (3 s.f.)

b Weight of the pyramid $= \frac{175}{3}\times 8.7 = 507.5$ grams

507.5 grams = 508 grams (correct to the nearest grams)

c EB is the hypotenuse of EOB.

$DB^2 = DA^2 + AB^2$

$DB^2 = 5^2 + 5^2$

$DB = \sqrt{50}$

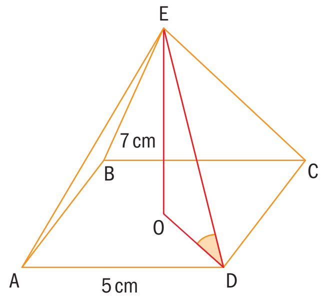
Now we find EB

$EB^2 = EO^2 + OB^2$

$EB^2 = 7^2 + \left(\frac{\sqrt{50}}{2}\right)^2$

$EB = \sqrt{61.5} = 7.842$ cm (4 s.f.)

d

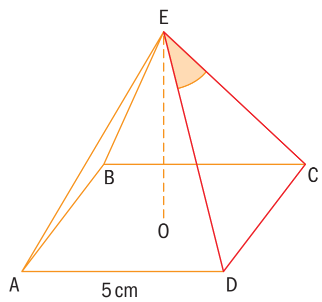


$\sin ODE = \dfrac{7}{7.842}$

$ODE = \sin^{-1}\left(\dfrac{7}{7.842}\right)$

$ODE = 63.2°$ (3 s.f.)

e



$\cos DEC = \dfrac{DE^2 + EC^2 - CD^2}{2\times DE\times EC}$

$\cos DEC = \dfrac{7.842^2 + 7.842^2 - 5^2}{2\times 7.842\times 7.842}$

$DEC = \cos^{-1}\left(\dfrac{7.842^2 + 7.842^2 - 5^2}{2\times 7.842\times 7.842}\right)$

$DEC = 37.2°$ (3 s.f.)

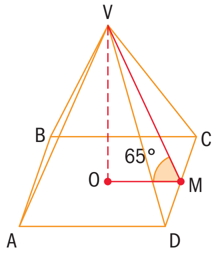
f Surface area = 4 × Area of DEC + Area of base

Surface area

$= 4\times\left(\frac{1}{2}\times 7.842\times 7.842\right)\times\sin 37.18° + 5^2$

Surface area = 99.3cm^2 (3 s.f.)

4 **a**



$\tan 65° = \frac{VO}{4}$

$VO = 4 \tan 65°$

$VO = 8.58$ cm (3 s.f.)

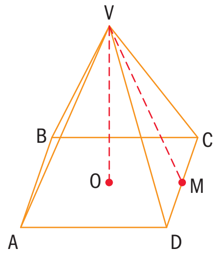
b **i** in triangle VMO

$\cos 65° = \frac{4}{VM}$

$VM = \frac{4}{\cos 65°}$

$VM = 9.46$ cm (3 s.f.)

ii $\angle DVC = 2 \times \angle MVC$



$\tan MVC = \frac{CM}{MV}$

$\tan MVC = \frac{4}{9.46}$

$MVC = \tan^{-1}\left(\frac{4}{9.46}\right)$

$MVC = 22.92....$

$DVC = 2 \times MVC$

$DVC = 2 \times 22.92...$

$DVC = 45.8°$

c Surface area of the pyramid = 4 × Area of DVC + Area of base

Surface area of the pyramid

$= 4 \times \left(\frac{1}{2} \times 9.46 \times 8\right) + 8^2$

Surface area of the pyramid = 215cm^2

d Volume of pyramid $= \frac{1}{3}(\text{Area of base} \times \text{Height})$

Volume of pyramid $= \frac{1}{3}(8^2 \times 8.58)$

Volume of pyramid = 183cm^3